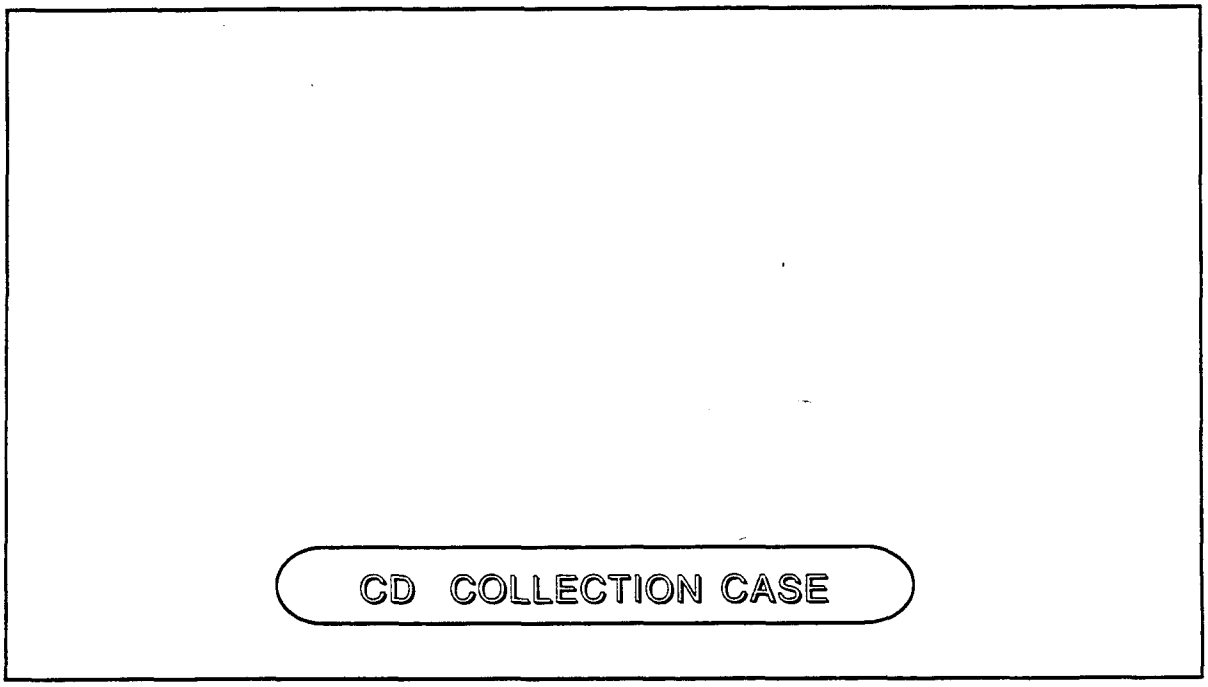
Collections B

The students in music class are putting together a collection of CDs. They are trying to learn about different kinds of music. All the collections have **jazz**, **classical**, and **rap**. Use markers to represent the different kinds of music and solve the problems.



1) Juan's collection:
   8 CDs in all
   2 rap
   2 more jazz than rap

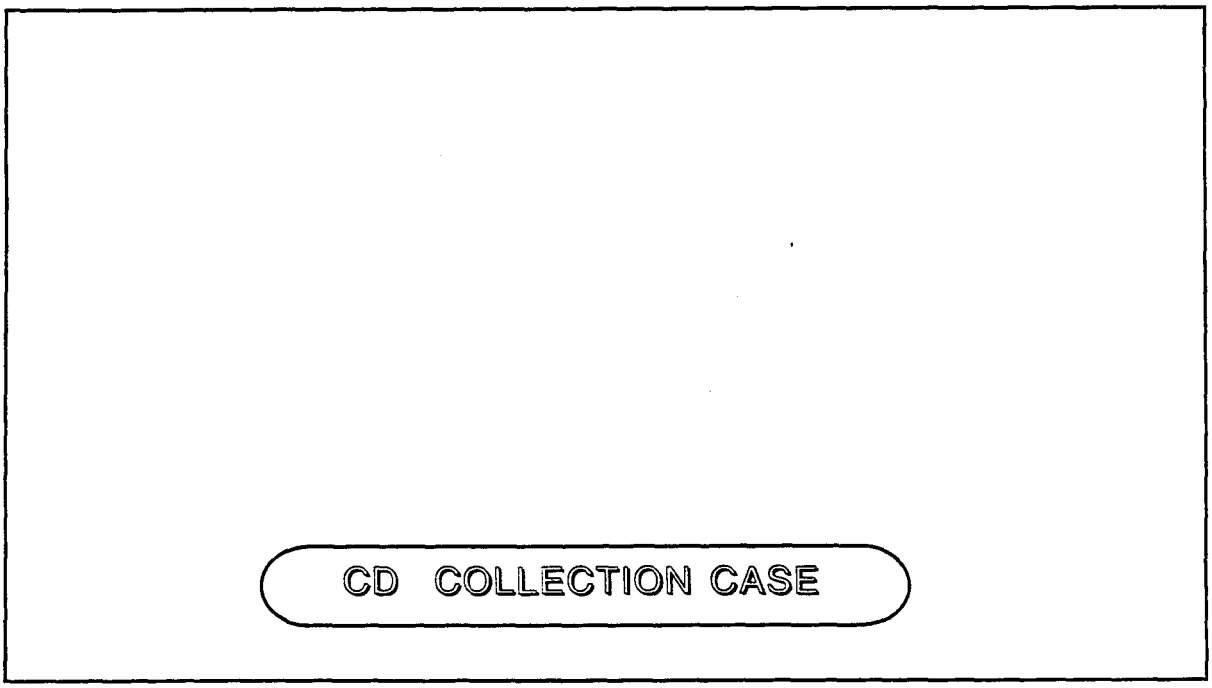
2) Mona's collection:
   5 classical
   3 fewer rap than classical
   10 CDs in all

3) Han's collection:
   4 jazz
   3 more classical than rap
   13 CDs in all

4) Omar's collection:
   15 CDs in all
   6 rap
   2 fewer rap than classical

Collections C

The students in music class are putting together a collection of CDs. They are trying to learn about different kinds of music. All the collections have **jazz**, **classical**, and **rap**. Use markers to represent the different kinds of music and solve the problems.



1) Ravinder's collection:
   2 rap
   twice as many jazz as rap
   9 CDs in all.

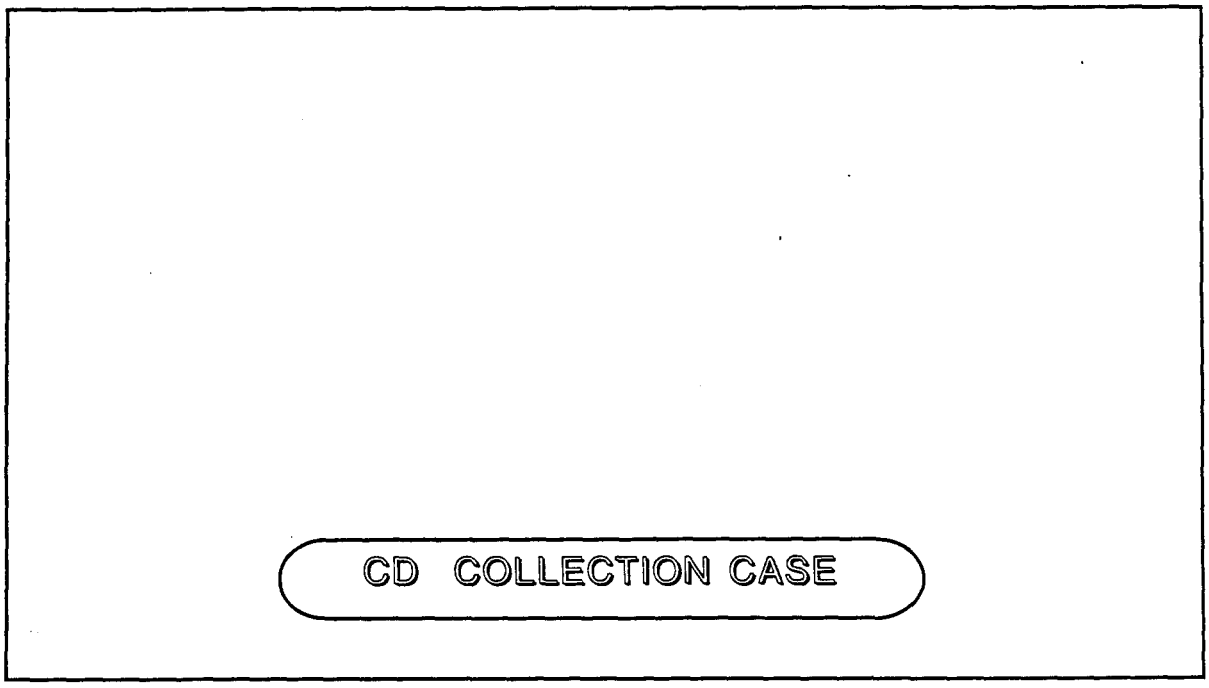
2) Andy's collection:
   14 CDs in all
   6 jazz
   1/2 as many classical as jazz

3) Ivan's collection:
   13 CDs in all
   4 classical
   twice as many rap as classical

4) Sun Je's collection:
   8 rap
   1/2 as many jazz as rap
   15 CDs in all

Collections D

The students in music class are putting together a collection of CDs. They are trying to learn about different kinds of music. All the collections have **jazz**, **classical**, and **rap**. Use markers to represent the different kinds of music and solve the problems.



1) Roberto's collection:
   2 rap
   3 times as many classical as rap
   10 CDs in all

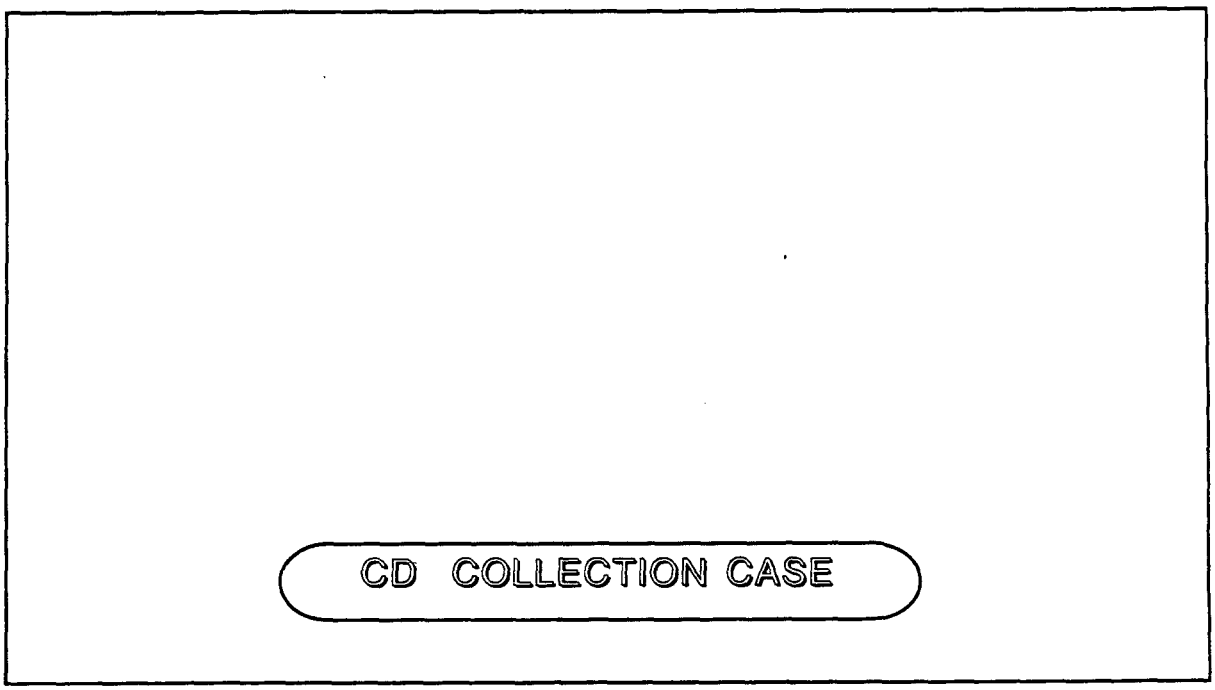
2) Jin Lee's collection:
   6 jazz
   1/3 as many rap as jazz
   12 CDs in all

3) Giovana's collection:
   3 classical
   4 times as many jazz as classical
   19 CDs in all

4) Dina's collection:
   12 rap
   1/4 as many jazz as rap
   20 CDs in all

Collections E

The students in music class are putting together a collection of CDs. They are trying to learn about different kinds of music. All the collections have **jazz**, **classical**, and **rap**. Use markers to represent the different kinds of music and solve the problems.



1) Kate's collection:
   8 CDs in all
   1/2 of the CDs are jazz
   1/4 of the CDs are rap

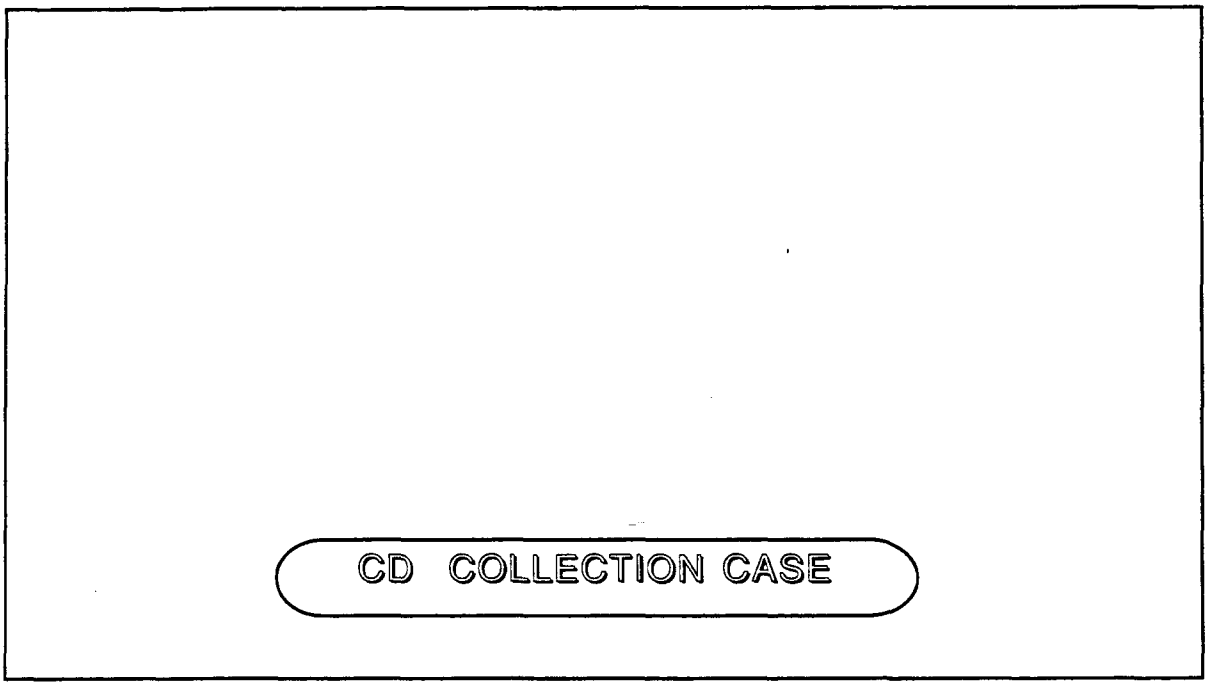
2) Daniel's collection:
   12 CDs in all
   1/4 of the CDs are jazz
   1/3 of the CDs are classical

3) Abdul's collection:
   6 CDs in all
   1/2 of the CDs are rap
   1/3 of the CDs are classical

4) Marta's collection:
   10 CDs in all
   1/2 of the CDs are jazz
   1/5 of the CDs are rap

Collections F

The students in music class are putting together a collection of CDs. They are trying to learn about different kinds of music. All the collections have **jazz**, **classical**, and **rap**. Use markers to represent the different kinds of music and solve the problems.



1) Kali's collection:
   6 CDs in all
   1 more jazz than classical
   1 more rap than jazz

2) Anna's collection:
   12 CDs in all
   1 more jazz than rap
   1 more rap than classical

3) Elena's collection:
   6 CDs in all
   1 fewer jazz than classical
   1 fewer rap than jazz

4) Hawa's collection:
   9 CDs in all
   1 fewer rap than classical
   1 fewer classical than jazz

Names _____

Date _____

Solutions to Collections

Collections A:

1. j =
   c =
   j + c + r =

   No. in Collection:
   j =
   c =
   r =

2. r =
   j =
   r + j + c =

   No. in Collection:
   r =
   j =
   c =

3. c =
   r =
   c + r + j =

   No. in Collection:
   c =
   r =
   j =

4. r =
   c =
   r + c + j =

   No. in Collection:
   r =
   c =
   j =

Collections B

1. j + c + r =
   r =
   j =

   No. in Collection:
   r =
   j =
   c =

2. c =
   r =
   c + r + j =

   No. in Collection:
   c =
   r =
   j =

3. j =
   c =
   j + c + r =

   No. in Collection:
   j =
   c =
   r =

4. c + j + r =
   r =
   c =

   No. in Collection:
   j =
   r =
   c =

Collections C

1. r =
   j =
   r + j + c =

   No. in Collection:
   r =
   j =
   c =

2. r + c + j =
   j =
   c =

   No. in Collection:
   r =
   j =
   c =

3. c + r + j =
   c =
   r =

   No. in Collection:
   j =
   c =
   r =

4. r =
   j =
   j + r + c =

   No. in Collection:
   r =
   j =
   c =

Collections D

1. r =
   c =
   r + c + j =

   No. in Collection:
   r =
   c =
   j =

2. j =
   r =
   j + r + c =

   No. in Collection:
   j =
   r =
   c =

3. c =
   j =
   c + j + r =

   No. in Collection:
   c =
   j =
   r =

4. r =
   j =
   c + j + r =

   No. in Collection:
   r =
   c =
   j =

Collections E

1. j + r + c =
   j =
   r =

   No.in Collection:
   j =
   r =
   c =

2. c + j + r =
   j =
   c =

   No. in Collection:
   c =
   j =
   r =

3. r + c + j =
   r =
   c =

   No. in Collection:
   r =
   c =
   j =

4. j + c + r =
   j =
   r =

   No. in Collection:
   c =
   j =
   r =

Collections F

1. c + r + j =
   j =
   r =

   No. in Collection:
   c =
   j =
   r =

2. c + r + j =
   j =
   r =

   No. in Collection:
   c =
   j =
   r =

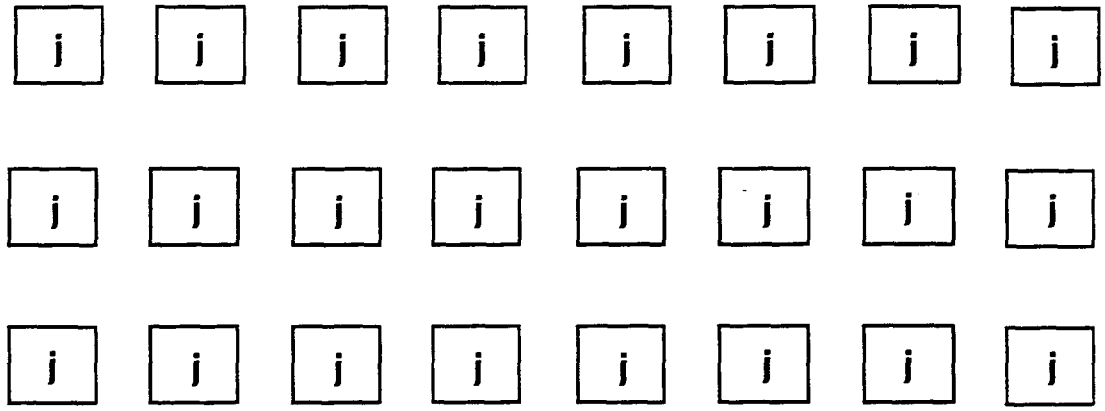
3. c + r + j =
   j =
   r =

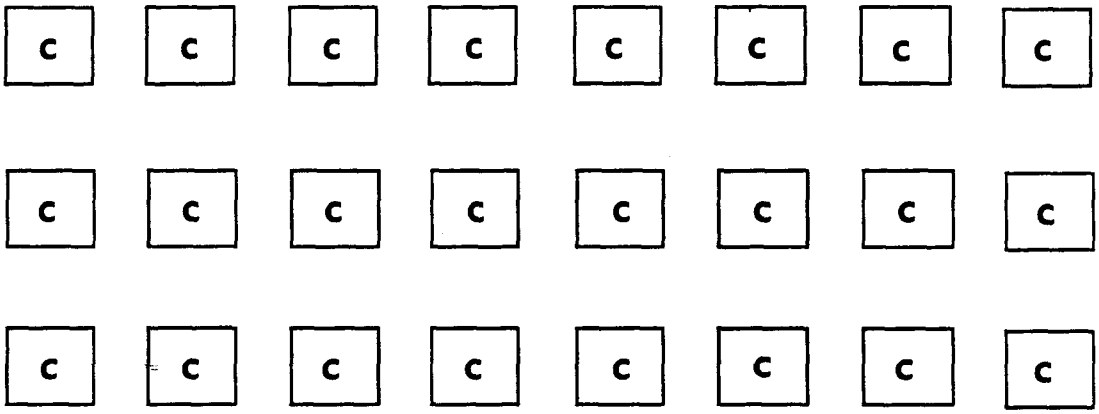
   No. in Collection:
   c =
   j =
   r =

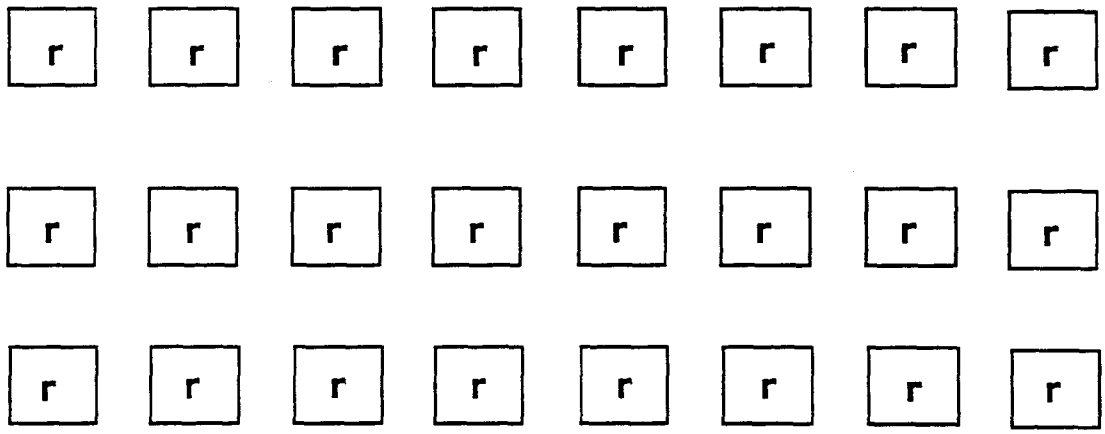
4. r + c + j =
   r =
   c =

   No. in Collection:
   j =
   r =
   c =

Collection Markers







Answer Key
Obj. 3

Collections A

1. $j = 2$
   $c = j + 1$
   $j + c + r = 9$
   No. in Collection:
   $j = 2$
   $c = 3$
   $r = 4$

2. $r = 3$
   $j = r - 1$
   $r + j + c = 6$
   No. in Collection:
   $r = 3$
   $j = 2$
   $c = 1$

3. $c = 4$
   $r = c + 1$
   $c + r + j = 11$
   No. in Collection:
   $c = 4$
   $r = 5$
   $j = 2$

4. $r = 6$
   $c = r - 1$
   $r + c + j = 14$
   No. in Collection:
   $r = 6$
   $c = 5$
   $j = 3$

Collections B

1. $j + c + r = 8$
   $r = 2$
   $j = r + 2$
   No. in Collection:
   $r = 2$
   $j = 4$
   $c = 2$

2. $c = 5$
   $r = c - 3$
   $c + r + j = 10$
   No. in Collection:
   $c = 5$
   $r = 2$
   $j = 3$

3. $j = 4$
   $c = r + 3$
   $j + c + r = 13$
   No. in Collection:
   $j = 4$
   $c = 6$
   $r = 3$

4. $c + j + r = 15$
   $r = 6$
   $c = r + 2$
   No. in Collection:
   $j = 1$
   $r = 6$
   $c = 8$

Collections C

1. $r = 2$
   $j = r + 2$
   $r + j + c = 9$
   No. in Collection:
   $r = 2$
   $j = 4$
   $c = 3$

2. $r + c + j = 14$
   $j = 6$
   $c = 1/2\ j$
   No. in Collection:
   $r = 5$
   $j = 6$
   $c = 3$

3. $c + r + j = 13$
   $c = 4$
   $r = 2c$
   No. in Collection:
   $j = 1$
   $c = 4$
   $r = 8$

4. $r = 8$
   $j = 1/2\ r$
   $j + r + c = 15$
   No. in Collection:
   $r = 8$
   $j = 4$
   $c = 3$

Answer Key
Obj. 3

Collections D

1. $r = 2$
   $c = 3r$
   $r + c + j = 10$
   No. in Collection:
   $r = 2$
   $c = 6$
   $j = 2$

2. $j = 6$
   $r = 1/3\ j$
   $j + r + c = 12$
   No. in Collection:
   $j = 6$
   $r = 2$
   $c = 4$

3. $c = 3$
   $j = 4c$
   $c + j + r = 19$
   No. in Collection:
   $c = 3$
   $j = 12$
   $r = 4$

4. $r = 12$
   $j = 1/4\ r$
   $c + j + r = 20$
   No. in Collection:
   $r = 12$
   $c = 5$
   $j = 3$

Collections E

1. $j + r + c = 8$
   $j = 1/2\ (8)$
   $r = 1/4\ (8)$
   No.in Collection:
   $j = 4$
   $r = 2$
   $c = 2$

2. $c + j + r = 12$
   $j = 1/4\ (12)$
   $c = 1/3\ (12)$
   No. in Collection:
   $c = 4$
   $j = 3$
   $r = 5$

3. $r + c + j = 6$
   $r = 1/2\ (6)$
   $c = 1/3\ (6)$
   No. in Collection:
   $r = 3$
   $c = 2$
   $j = 1$

4. $j + c + r = 10$
   $j = 1/2\ (10)$
   $r = 1/5\ (10)$
   No. in Collection:
   $c = 3$
   $j = 5$
   $r = 2$

Collections F

1. $c + r + j = 6$
   $j = c + 1$
   $r = j + 1$
   No. in Collection:
   $c = 1$
   $j = 2$
   $r = 3$

2. $c + r + j = 12$
   $j = r + 1$
   $r = c + 1$
   No. in Collection:
   $c = 3$
   $j = 5$
   $r = 4$

3. $c + r + j = 6$
   $j = c - 1$
   $r = c + 1$
   No. in Collection:
   $c = 2$
   $j = 1$
   $r = 3$

4. $r + c + j = 9$
   $r = c - 1$
   $c = j - 1$
   No. in Collection:
   $j = 4$
   $r = 2$
   $c = 3$

Objective 4: Evaluate expressions using substitution.

Vocabulary

value
evaluate
substitute
substitution
simplify

Materials

Transparencies

Evaluating Expressions Using Substitution
Evaluating Expressions

Student Copies

Evaluating Expressions Using Substitution
Evaluating Expressions
Can't Wait to Evaluate
Vocabulary Review

Language Foundation

1. Ask students if they are familiar with the word value. Discuss the meaning of the word in its various contexts. For example, one meaning is how important or useful something is. Ask students if they have something from a grandparent that has great value to them. Students might also think of value as the amount of money something is worth. On the other hand, they might think of something on sale as a "good value." Brainstorm other meanings of the word such as moral values. Then, explain to students that in math, the term **value** is used with variables and expressions. It denotes how much a variable is and helps determine the answer when evaluating an expression.

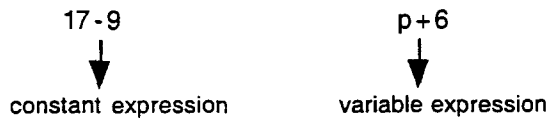
2. Discuss the word evaluate. Explain that evaluate means to consider something or someone in order to make a judgment. It usually means to think carefully about something. Explain that in math the meaning of evaluate is very specific. Explain that to **evaluate** an expression there must be an actual answer.

3. Ask students who teaches the class when a teacher is absent. They should be familiar with the word substitute teacher, the person who takes his or her place. The substitute teacher takes the place of the classroom teacher. **Substitution** is the process of putting one thing (or person) in the place of another. Explain to students that in this lesson they will use substitution to find the total value of expressions.

4. Relate **simplify** to the word simple. Explain that simplify means to make something clearer or easier to do or understand. Ask students for ways they can simplify their lives. For example, they could use a calculator to do computations quicker and save time. Explain that in math, **simplify** means to combine terms to make computation easier.

Mathematics Component

Warm-up: Write the following on the board:



Ask students to tell how these two expressions are **different**. (The constant expression 17 - 9, also called a **numerical expression**, contains **only** constants and an operation symbol. The variable expression p + 6, also called an **algebraic expression**, contains a variable and a constant along with an operation symbol.) Have students practice writing other examples of constant expressions and variable expressions on the board.

1. Define the terms "value" and "evaluate."
   - Review the everyday meaning of the words value and evaluate as described in the language foundation.
   - Explain that in math to **evaluate** an expression means to find its total **value**.
   - Write the following constant expression on the board: **6 + 5**.
   - Say, "To **evaluate** the expression 6 + 5, we need to find its **total value**. Elicit that the total value of the expression is 11. Therefore, when you evaluate 6 + 5, the correct response is 11."

2. Evaluate expressions using substitution.
   - Write the following variable expression on the board: **a + 4**.
   - Ask students to evaluate the expression **a + 4**. If students need help understanding "evaluate" say, "What is the total value of the expression?" Lead them to understand that they can't evaluate this expression because they don't know the value of **a**.
   - Review the meaning of the word substitute given in the language foundation. Explain that if they know the value of the variable, they can take the variable out of the expression and substitute its value. This is called **substitution** because the value is taking the place of the variable.
   - Point back to the expression **a + 4** and say, "What if the value of **a** is **2**. Can we evaluate the expression **a + 4**?" (Yes, it is 6.) Ask why the answer is 6. (The total value is 6 since 2 + 4 = 6.)
   - Write several expressions on the board, assign a value to the variable, and then have students help you use substitution to rewrite and then evaluate each expression. Reinforce the words **substitution** and **evaluate** as you write each expression.
   - Ask students to explain orally why the variable must be replaced before evaluating an expression. (The value of the variable is unknown unless it is stated; therefore, they cannot find the total value of the expression.) Lead students to understand that by replacing the variable with a specific value, they can "evaluate" the expression.

- Write **a + 4** on the board again and ask students to evaluate the expression if **a = 5** (9) and then again if **a = 3**. (7) Ask, "Will the total value of this expression always be the same? Why?" (No, because **a** is a variable and it can have different values.)
- Write the expression **b - 4** on the board. Model using substitution to evaluate the expression with different values for **b** as follows:

b - 4; **b** = 8	**b** - 4; **b** = 10	**b** - 4; **b** = 30
8 - 4	10 - 4	30 - 4
4	6	26

Remind students that in an expression, as the value of the variable changes then the value of the expression changes.

- Review the steps to follow when evaluating an expression. Write them on the board.
  1. **Copy** the expression and **write** the value of the variable.
  2. **Substitute** by replacing the variable(s) with the value(s).
  3. **Simplify** the expression showing one computation per line.

3. Create a list of steps used to evaluate expressions using substitution.
   - Use the transparency Evaluating Expressions Using Substitution.
   - Do the first problem from the worksheet on the overhead as the students record their work on their answer sheet. Create a shortened list of steps as follows:

1)	10 **a**; **a** = 5	(COPY)
	10 (5)	(SUBSTITUTE)
	50	(SIMPLIFY)

   - Do problem 15 together as an example of a problem with more than one computation:

15)	2 + **b** - **a**; **a** = 5 & **b** =14	(COPY)
	2 + 14 - 5	(SUBSTITUTE)
	16 - 5	(SIMPLIFY)
	11	(SIMPLIFY)

   - Do additional problems together as needed. If necessary, review the commutative property of multiplication for problem 4. Remind students that the order of the multiplication will not affect the answer. Have students work independently or in pairs to evaluate the remaining problems. This may be a good opportunity for students to practice using calculators to check their finished work.

4. Practice the steps used to evaluate expressions using substitution.
   - Provide students with copies of Evaluating Expressions.
   - Work through at least a few expressions together. Model how to substitute the value of each

variable given in the first column into the five different expressions at the top. Have students record their work on notebook paper and then transfer their answers to the appropriate cells on the activity page. For example, in the first table the value of the variable in the first row is w =1. Students should show the following work on notebook paper for the first two expressions:

Expression1			Expression2		
2 **w**;	**w** = 1	(COPY)	5 - **w**;	**w** = 1	(COPY)
2 (1)		(SUBSTITUTE)	5 - 1		(SUBSTITUTE)
2		(SIMPLIFY)	4		(SIMPLIFY)

- The answers "2" and "4" should then be recorded in the first two horizontal cells. Model other cells, as needed.

Value of: **w**	**2w**	**5 - w**	**3w + 1**	**4w - 2**	$\frac{5}{w}$
1	2	4			
2					
2.5					
$\frac{1}{2}$					

- Have students complete each table showing their work on notebook paper and recording their answers in the appropriate cells. Go over and discuss student responses.
- The activity sheet Can't Wait to Evaluate is provided to review evaluating algebraic expressions.

Language Development Activities

- Notebook

  Have students set up a notebook for taking notes or recording essential concepts. Keeping a notebook will help students develop good study and organizational skills and will provide them with a resource for review. It will also provide a resource that they can take with them to subsequent math classes. Ask students to write the steps for evaluating an expression in their notebook:

  1. **Copy** the expression and **write** the value of the variable.
  2. **Substitute** by replacing the variable(s) with the value(s).
  3. **Simplify** the expression showing one computation per line.

  Have students include examples of each step in their notes.

- Vocabulary Reinforcement

  The activity page Vocabulary Review will provide additional reinforcement of new terms introduced in this objective as well as review terms introduced earlier.

- Writing Prompt

  To reinforce the concept of simplifying expressions have students complete the writing prompt in Part II of the Vocabulary Review activity page. When students write the steps in simplifying an expression, remind them to include transitions such as first, then, and last.

Additional Resources

The Algebra Lab, Middle School, Lesson 2 , Activity 2

Name _____ (Transparency/Student Copy)

Evaluating Expressions Using Substitution

For each expression below a = 5, b = 14, and c = 6.

1) 10a

2) 27 - c

3) a + 5 + b

4) abc

5) a + 13

6) 3ac

7) c - a

8) a + 24

9) 5c - b

10) 5b + 5c

11) b - 2

12) 5ac

13) $\frac{c}{3}$

14) 8 + c

15) 2 + b - a

16) $\frac{25}{a}$

17) $\frac{b}{7}$

18) 6a + b

19) 2b - 2a

20) 9 + c - 2

Name _____ (Transparency/Student Copy)
Date _____

Evaluating Expressions

Complete the tables by evaluating each expression.

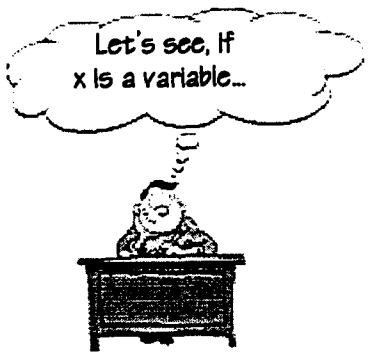
Value of: w	2w	5 - w	3w + 1	4w - 2	$\frac{5}{w}$
1					
2					
2.5					
$\frac{1}{2}$					

Value of: x	3x	x + 5	3 + 2x	$\frac{x}{2}$	4x + 2x
2					
6					
0.6					
$\frac{2}{3}$					

Value of: y	10y	y	2y + y	5y - 1	$\frac{y}{2}$
10					
$\frac{3}{4}$					

Name: _____

Can't Wait To Evaluate



Match:

____ 1)	25 more than y	A) $25 - y$
____ 2)	25 less than y	B) $y - y$
____ 3)	y less than 25	C) $y + 25$
____ 4)	y more than y	D) $y + y$
____ 5)	y less than y	E) $y - 25$

Evaluate the expression

1) $10 + b$ when $b = 8$

10 + 8 = 18

2) $6\ z$ when $z = 20$

6 x 20 = _____

3) $130 - w$ when $w = 50$

_____ - _____ = _____

4) $\frac{28}{r}$ when $r = 7$

_____ ÷ _____ = _____

5) $12 + k$ when $k = 3.5$

_____ + _____ = _____

6) $6d - 60$ when $d = 11$

_____ - _____ = _____

Evaluate the expression $250 - n$ for the following values of n:

1) $n = 50$ 2) $n = 120$ 3) $n = 75$ 4) $n = 248$

_____ _____ _____ _____

Can't Wait To Evaluate

Evaluate each expression for $a = 244$ and $b = 65$

1) $a - 40$

2) $(a + b) - 20$

3) $(365 + b) - a$

4) $a - b$

5) $(b + a) + b$

6) $\frac{a+b}{3}$

Complete the chart.

Expression	What the Expression Means	Value of the Variable	Evaluate the Expression
$5h$	____ times the number ____	$h = 10$	$5 (\_\_\_) = \_\_\_$
$\frac{m}{11}$	the quotient of a number ____ and ____	$m = 88$	$\frac{\square}{11} = \_\_\_$
$\frac{1}{4}c$	one fourth of a number ____	$c = 100$	$\frac{1}{4} (\quad) = \_\_\_$
$r + 82$	____ more than a number ____	$r = 18$	$\_\_\_ + 82 = \_\_\_$
$27.8 - x$	the difference between ____ and a number ____	$x = 4.3$	$27.8 - \_\_\_ = \_\_\_$
$t - 30$	____ less than a number ____	$t = 560$	$\_\_\_ - 30 = \_\_\_$
$900 \div j$	____ divided by a number ____	$j = 6$	$900 \div \_\_\_ = \_\_\_$

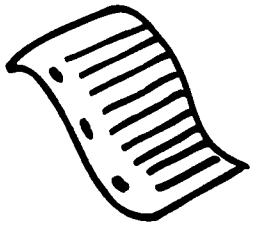
Name _____

Vocabulary Review

Part I. Sentence Completions. Complete the sentences with a math term in the box. You will need to use a few words more than one time.

constant	expression	substitution
evaluate	simplify	variable

1. The number **23** in the expression 5y + 23 is called a(n) _____.
2. 6x + 2 is an example of a(n) _____ expression.
3. You are using _____ when you put a number in the place of a variable to find the value of an expression.
4. To _____ an expression you must find its total value.
5. When you _____ an expression, you combine terms to make it easier to find the answer.
6. The letter **b** in the expression 5 + b + 6 is called a(n) _____.
7. A(n) _____ includes a constant or variable **and** an operation.
8. 65 + 35 is an example of a(n) _____ expression.



Part II. Writing about Math. Think about the way you simplify expressions in math. Explain in words how to simplify an expression. Use words like <u>first</u>, <u>then</u>, and <u>last</u> in your response.

Answer Key - Objective 4

Evaluating Expressions Using Substitution

1) $10\mathbf{a}$; $\mathbf{a} = 5$
$10\ (5)$
50

2) $27 - \mathbf{c}$; $\mathbf{c} = 6$
$27 - 6$
21

3) $\mathbf{a} + 5 + \mathbf{b}$; $\mathbf{a} = 5$, $\mathbf{b} = 14$
$5 + 5 + 14$
$10 + 14$
24

4) $\mathbf{abc}$; $\mathbf{a} = 5$, $\mathbf{b} = 14$, $\mathbf{c} = 6$
$(5)(14)(6)$
$(70)(6)$
(420)

5) $\mathbf{a} + 13$; $\mathbf{a} = 5$
$5 + 13$
18

6) $3\mathbf{ac}$; $\mathbf{a} = 5$, $\mathbf{c} = 6$
$(3)(5)(6)$
$(15)(6)$
90

7) $\mathbf{c} - \mathbf{a}$; $\mathbf{c} = 6$, $\mathbf{a} = 5$
$6 - 5$
1

8) $\mathbf{a} + 24$; $\mathbf{a} = 5$
$5 + 24$
29

9) $5\mathbf{c} - \mathbf{b}$; $\mathbf{b} = 14$, $\mathbf{c} = 6$
$(5)(6) - 14$
$30 - 14$
16

10) $5\mathbf{b} + 5\mathbf{c}$; $\mathbf{b} = 14$, $\mathbf{c} = 6$
$(5)(14) + (5)(6)$
$70 + 30$
100

11) $\mathbf{b} - 2$; $\mathbf{b} = 14$
$14 - 2$
12

12) $5\mathbf{ac}$; $\mathbf{a} = 5$, $\mathbf{c} = 6$
$(5)(5)(6)$
$(25)(6)$
150

13) $\frac{\mathbf{c}}{3}$; $\mathbf{c} = 6$
$\frac{6}{3}$
2

14) $8 + \mathbf{c}$; $\mathbf{c} = 6$
$8 + 6$
14

15) $2 + \mathbf{b} - \mathbf{a}$; $\mathbf{a} = 5$, $\mathbf{b} = 14$
$2 + 14 - 5$
$16 - 5$
11

16) $\frac{25}{\mathbf{a}}$; $\mathbf{a} = 5$
$\frac{25}{5}$
5

17) $\frac{\mathbf{b}}{7}$; $\mathbf{b} = 14$
$\frac{14}{7}$
2

18) $6\mathbf{a} + \mathbf{b}$; $\mathbf{a} = 5$, $\mathbf{b} = 14$
$(6)(5) + 14$
$30 + 14$
44

19) $2\mathbf{b} - 2\mathbf{a}$; $\mathbf{a} = 5$. $\mathbf{b} = 14$
$(2)(14) - (2)(5)$
$28 - 10$
18

20) $9 + \mathbf{c}\ 2$; $\mathbf{c} = 6$
$9 + 6 - 2$
$15 - 2$
± 3

Answer Key
Objective 4

Evaluating Expressions

Complete the tables by evaluating each expression.

Value of: w	2w	5 w	3w + 1	4w 2	$\frac{5}{w}$
1	2	4	4	2	5
2	4	3	7	6	2 1/2
2.5	5	2.5	8.5	8	2
$\frac{1}{2}$	1	4 1/2	2 1/2	0	10

Value of: x	3x	x + 5	3 + 2x	$\frac{x}{2}$	4x + 2x
2	6	7	7	1	12
6	18	11	15	3	36
0.6	1.8	5.6	4.2	.3	3.6
$\frac{2}{3}$	2	5 2/3	4 1/3	1/3	4

Value of: y	10y	y	2y + y	5y - 1	$\frac{y}{2}$
10	100	10	30	49	5
$\frac{3}{4}$	7 1/2	3/4	2 1/4	2 3/4	3/8

Can"t Wait to Evaluate

Match

1) C
2) E
3) A
4) D
5) B

Evaluate the Expression

2) 120
3) 130 - 50 = 80
4) 28 ÷ 7 = 4
5) 12 + 3.5 = 15.5
6) 66- 60 = 6

Evaluate the Expression for 250 - n

1) 250 - 50 = 200
2) 250 - 120 = 130
3) 250 - 75 = 175
4) 250 - 248 = 2

Evaluate Each Expression for a = 244 and b = 65

1) 244 - 40 = 204
2) (244 + 65) - 20 = 289
3) (365 + 65) -244 = 186
4) 244 - 65 = 179
5) (65 + 244) + 65 = 374
6) 309 ÷ 3 = 103

Complete the Chart

1) 5 times the number h ; 5(10) = 50
2) the quotient of a number m and 11; 88 /11 = 8
3) one fourth of a number c ; 1/4 (100) = 25
4) r more than a number 82; 18 + 82 = 100
5) the difference between 27.8 and a number x; 27.8 - 4.3 = 23.5
6) thirty less than a number t ; 560 - 30 = 530
7) 900 divided by a number j; 900÷ 6 = 150

Vocabulary Review

1) constant
2) variable
3) substitution
4) evaluate
5) simplify
6) variable
7) expression
8) constant

Objective 5: Define equation. Represent unknowns using variables in expressions.

Vocabulary

expression
equation
substitute

Materials

calculators

Transparencies

Variable Expressions #1
Variable Expressions #2

Student Copies

Equations and Expressions
Writing about Math

Language Foundation

1. Review the terms expression, substitute, constant, and variable.

2. Ask students if they know what the term equation means. Have a student write an example of an equation on the board. Explain to students that an **equation** in math is a statement that uses the equal sign (=) to show that two quantities have the same value or are equal. Point out that the word equation has part of the word equal in it: **equa**tion.

3. Explain to students that they can use any of the following to represent the equal sign (=): "equals," "is," and "is equal to."

4. Pointing out the word "igual" in Spanish will help Spanish speakers make a connection between the word "igual" (equal) in Spanish and the meaning of the words equal and equation in English.

Mathematics Component

1. Define the word equation.
   - Place the transparency Variable Expressions #1 on the overhead projector. Use a cover sheet to help students focus on one part of the transparency at a time.
   - Explain that this transparency shows an advertisement for a CD store. Uncover the information in the box and read each line aloud.
   - Read the questions below the box one at a time and ask different students to give the answer to a question and then tell how they found the answer. Record students' responses as shown below.

	Amount	How?
1. How much does 1 CD cost at this store?	$13	13 x 1 = 13
2. How much would 2 CDs cost ?	$26	13 x 2 = 26
3. How about 3 CDs?	$39	13 x 3 = 39
4. How about 50 CDs?	$650	13 x 50 = 650

   - Point to and tell students that each number sentence written in the column labeled "How?" is called an **equation.** Explain that an **equation** is a statement that two quantities are equal. The left-hand side is equal to the right-hand side. Tell students that equations can be number sentences such as 13 x 1 = 13, or they may have variables in them such as p + 2 = 9.

2. Represent an unknown with a variable expression. (Continue using the transparency from activity 1 above.)
   - Take this opportunity to remind students that multiplication can be shown in at least three different ways such as: 13 x 2, (13)(2), or 13 • 2. Explain that the **equations** recorded on the transparency could be written using any of these methods.
   - Ask students if there is a way to find the amount for any number of CDs. Lead students to see that the amount spent on CDs is always the number of CDs multiplied by 13.
   - Tell students that since the number of CDs changes, a variable can be used to represent the number of CDs. Say, "We will let the variable **n** represent the number of CDs." Write **n** as the variable on the transparency.
   - Ask, "What expression can we write to represent the amount for any number of CDs?" (13n) Write 13n on the transparency as shown below.

     Amount for any number of CDs: **13n**
   - Have students explain whether "13n" is a constant expression or a variable expression and why. (It is a variable expression because it contains at least one variable along with an operation. Remind students that when a constant is written beside a variable they should multiply.)
   - Go over the example at the bottom of the transparency. Remind students that the **variable expression** 13n is used to help find the amount of money needed to buy 36 CDs. The number of CDs (36) is **substituted** for the variable **n** in the expression.

- Explain that the **equation** (13)(36) = $468 shows the total amount for the CDs and how the amount was calculated.
- Remind students that an **equation** is a statement that two quantities are equal. Ask students to name the two quantities which are equal in the equation (13)(36) = $468. (Thirteen times thirty-six is equal to 468.)

3. Reinforce the definition of equation and the concept of representing unknowns using variable expressions.
   - Place the transparency Variable Expressions #2 on the overhead projector. Use a cover sheet to help students focus on one part of the transparency at a time. Do this activity in the same way as activity 2 above.
   - Explain that this transparency shows an advertisement for a clothing store. Uncover the information in the box and read each line aloud.
   - Read the questions below the box one at a time and ask different students to give the answer to a question and then give an **equation** that tells how they found the answer. Record students' responses as shown below. (Note: This is a good opportunity to introduce the concept that "of" means multiply in mathematics and that multiplying by 1/4 is the same as dividing by 4.)

	Amount Saved	**Equation**
1. How much do you save on a $12 bill? ?	$3	12/4 = 3 or 1/4(12)
2. How much do you save on a $16 bill ?	$4	16/4 = 4 or 1/4(16)
3. How much do you save on a $20 bill?	$5	20/4 = 5 or 1/4(20)
4. How much do you save on a $124 bill?	$31	124/4 = 31 or 1/4(124)

   - Ask students if there is a way to find the amount saved on any total bill. Lead students to see that the amount saved is always the total bill divided by 4 or multiplied by 1/4.
   - Tell students that since the total bill changes, a variable can be used to represent the total bill. Say, "We will choose a variable to represent the total bill." Have a student suggest a variable and record it in the space provided on the transparency. (The variable t might be suggested.)
   - Ask, "What expression can we write to represent the amount saved on any total bill" (t/4) Write t/4 on the transparency as shown below.

     Amount saved on any total bill: t / 4 or 1/ 4t

   - Have students look at the example at the bottom of the transparency. Call on different students to answer the following questions about the example.
     - What **variable expression** is used to help find the amount saved on any bill? (t / 4 or 1/4t)
     - What number is **substituted** for the variable **t**? (32)
     - What **equation** shows the total amount saved and also how the answer was calculated? (32/4 = $8 or 1/4(32) = $8)

4. Reinforce the concepts of equations and expressions.
   - The activity sheet Equations and Expressions is included for additional practice. Have students complete this activity independently or in pairs.
   - Check answers together by having different students come up to the overhead and write their answers. The chart on page 1 of the activity sheet provides a good opportunity to review vocabulary introduced in Objective 4. Ask students whether each equation is an **algebraic equation** or a **numerical equation** and have them explain why. Also, for each expression, have students tell whether it is a **constant expression** or a **variable expression**.

Language Development Activities

- Writing Prompt

  To help students internalize the concepts of equations and expressions, have students complete the activity sheet Writing about Math.

	Amount	How?
1. How much does 1 CD cost?	_____	_____
2. How much do 2 CDs cost ?	_____	_____
3. How much do 3 CDs cost?	_____	_____
4. How much do 50 CDs cost?	_____	_____

Let _____ represent the number of CDs purchased.

Amount for <u>any</u> number of CDs: _____

EXAMPLE - For 36 CDs the amount would be:

13 n

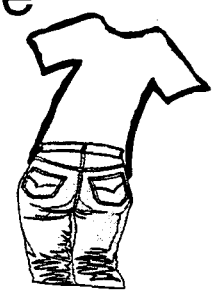
▼ ▼

(13)(36) = $468.00

Maria's Clothing Store

Save $\frac{1}{4}$ every time on your total bill!

Sunnydale Mall



	Amount Saved	Equation
1. How much do you save on a $12 bill?	_____	_____
2. How much do you save on a $16 bill?	_____	_____
3. How much do you save on a $20 bill?	_____	_____
4. How much do you save on a $124 bill?	_____	_____

Let _____ represent the total bill.

Amount saved on <u>any</u> total bill: _____

EXAMPLE - Amount saved on a $32 total bill:

$t \div 4$ or $\frac{1}{4}t$

$32 \div 4 = \$8$ $\frac{1}{4}(32) = \$8$

Name _____

Equations and Expressions

An equation is a statement that two quantities are equal. An expression is a combination of constants and/or variables along with an operation.

Is each problem an equation or an expression? Place a check mark (√) in the correct column.

Which is it?	Equation	Expression
1. 5 + 6 = 11		
2. 15 - r = 20		
3. 12s		
4. 20 - 5 = 10 + 5		
5. 3r = m - k		
6. p(5 - 4)		
7. 9y + 6z - 11		
8. 12/4 + 2/3 - 1/8		
9. 15 - 3 = 1/3s		
10. 26 = p - r		
11. 22 = 45 - 23		
12. 6d - 4t = 9 - m		
13. x/4		
14. 14 = 2(4 + 3)		
15. 100 - 25 = 75		

Can you write an equation to help solve each of the following problems?

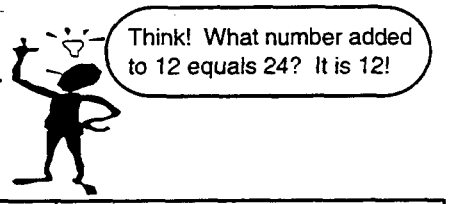
HINT: Represent each number with a variable until you have solved the problem.

Example: I am thinking of a number. If you add 12, you get 24. What is my number?

n

n + 12 = 24

12

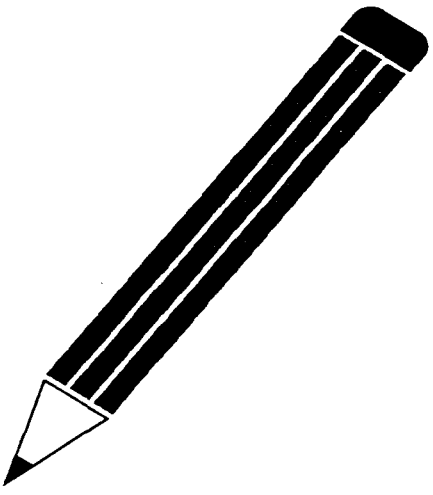


1. I am thinking of a number. If I subtract 10 from the number, I get 30. **Variable:** I will use _____ **Equation:** _____ **Solution:** _____	2. I am thinking of a number. If I multiply the number by 4, I get 24. **Variable:** I will use _____ **Equation:** _____ **Solution:** _____	3. I am thinking of a number. If I divide the number by 3, I get 27. **Variable:** I will use _____ **Equation:** _____ **Solution:** _____
4. I am thinking of a number. If I add 6 and the number, I get 13. **Variable:** I will use _____ **Equation:** _____ **Solution:** _____	5. I am thinking of a number. If I multiply 2 and the number, I get 18. **Variable:** I will use _____ **Equation:** _____ **Solution:** _____	6. I am thinking of a number. If I divide the number by 4, I get 36. **Variable:** I will use _____ **Equation:** _____ **Solution:** _____
7. I am thinking of a number. If I subtract 3 and then subtract 3 again, I get 9. **Variable:** I will use _____ **Equation:** _____ **Solution:** _____	8. I am thinking of a number. If I take half of the number, I get 10. **Variable:** I will use _____ **Equation:** _____ **Solution:** _____	9. I am thinking of a number. If I add the number two times and then subtract 6, I get 10. **Variable:** I will use _____ **Equation:** _____ **Solution:** _____

Name _____

Writing About Math

What is the difference between an expression and an equation? Give examples when you write your answer.



Answer Key

Intro to Algebra- Obj. 5

Equations and Expressions

1) Equation - numerical
2) Equation - algebraic
3) Expression - variable (or algebraic)
4) Equation - constant (or numerical)
5) Equation - variable
6) Expression - variable (or algebraic)
7) Expression - variable (or algebraic)
8) Expression - constant (or numerical)
9) Equation - variable (or algebraic)
10) Equation - variable
11) Equation - constant (or numerical)
12) Equation - variable (or algebraic)
13) Expression - variable(or algebraic)
14) Equation - numerical (or constant)
15) Equation - numerical (or constant)

p. 2

Students may choose any letter as a variable.

1) n ; n -10 = 30 ; n=40
2) n; 4n = 24 ; n=6
3) n; n/3=27 ; n = 81
4) n; 6 + n = 13 ; n = 7
5) n; 2n = 18 ; n = 9
6) n; n/4 = 36 ; n = 144
7) n; n - 3 - 3 = 9; n= 15
8) n; n/2 or 1/2(n) = 10; n= 20
9) n; n + n - 6 =10 ; n = 8

Objective 6: Combine like terms to simplify expressions

Vocabulary

term
coefficient
like term
unlike term
identical
combine
simplify

Materials

colored overhead markers
2 erasers, 2 pencils
markers or colored pencils

Transparencies

- Identifying Like and Unlike Terms and Simplifying

Student Copies

- Identifying Like and Unlike Terms and Simplifying
- Working with Terms
- Simplify by Combining Terms
- Vocabulary Review

Language Foundation

1. Discuss the word term. Explain that this word has different meanings in English. It can mean part of something, such as a period of time. For example, a short term is a short period of time. It can refer to a part of the school year or a term in office such as the time a governor, senator, or president is elected to serve. Remind students of another definition: a word or expression that has a particular meaning. They should be familiar with the word term as it applies to the vocabulary words they learn in their science classes and words they have been learning in math. Explain that in this lesson **term** has a more specific meaning: it is a part of a mathematical expression.

2. Introduce and/or review prefixes with students. Point out that a prefix is a word part that comes before a word and can change the meaning of a word. Teach them the prefix **co** which means "with" or "together". Explain that in this lesson students will learn the meaning of a **coefficient**. Knowing the meaning of the prefix **co** should help them understand the meaning of **coefficient** in terms of math.

3. Discuss the meaning of the word like which has many meanings in English. Brainstorm the various meanings with the class. Students are probably most familiar with two definitions: "similar in some way"; and "to enjoy something or think it is good." Explain that in this lesson the word **like** means "the same" as in **like terms**. Introduce the word **unlike** and explain that **unlike** is the opposite of **like** and means something is different.

4. Ask if students know the meaning of the word identical or if someone knows any identical twins. Tell students that the word identical means "exactly the same." Brainstorm examples of things that are identical such as a pair shoes or a pair of earrings. Tell students that when they look at math terms, they will need to identify terms that are **identical**. These must be exactly the same.

Mathematics Component

1. Define terms.
   - Review the meaning of the word **term** as described in the language foundation.
   - Explain that, in mathematics, **term** refers to a section or part of a mathematical expression.
   - A term can be a constant (6), a variable (y), or a product or quotient of constants and/or variables.

     Examples of terms:

     Constants - 4, 27, 9...

     Variables - a, x, z...

     Product of constants and/or variables - 3b; 22y, ab, 7xy...

     Quotient of constants and/or variables - $\frac{12}{4}$, $\frac{9}{a}$, $\frac{c}{d}$ ...

2. Identify terms.
   - Distribute individual copies of the activity sheet Identifying Like and Unlike Terms and Simplifying. Tell students that they will go through each problem and underline the terms.
   - Use a transparency copy of the activity sheet to model. Point to problem 1 on the activity sheet and identify the four terms. (1, 1, 1, and 25) Underline each term with black marker as it is identified. (See the underlined terms on the answer key.)
   - Ask students to identify the operation symbols between the terms.
   - Work several problems as a class and then have students work with a partner to underline the terms in the remaining problems.
   - When students have finished all problems, have them model solutions on the overhead.

3. Identify coefficients.
   - Now go back to problem 4 on the same transparency used above and review the four underlined terms. (2x, x, y, 2y) Explain that in the term 2x, the 2 is called the **coefficient**. Say, "The **coefficient** is the number in front of the variable(s)."
   - Point to the term 2y in the same problem and ask what the coefficient is? (2)
   - Point to the terms x and y in the same problem and ask what the coefficients are. (Students may think that there is no coefficient.) Explain that if a coefficient is not written in front of a variable, it is assumed to be 1. The coefficients of x and y in problem 4 are, therefore, 1.
   - Go back and look at additional terms which include a constant and a variable. Name the coefficient of several terms. Be sure that students are comfortable with the term coefficient and able to identify them in an expression.

4. Explore the concepts of like and unlike.
   - Place 2 erasers, spaced apart, on a desk where all students can see them.
   - Say to students, "Tell me what is on the desk." (Most students will say 2 erasers. If they just answer "erasers," lead them by asking them to tell you more.)

- Ask students if anyone answered, "You have 1 eraser and 1 eraser." Ask why most responded, "You have 2 erasers." Lead students to understand that because the items are alike, it is easier to combine them and say, "You have 2 erasers."
- Place 1 eraser and 1 book on the desk and repeat the same procedure. Say, "Tell me what is on the desk." (The students should say 1 book and 1 eraser.) Explain that when the items are not alike, they may not be combined when we name them. For 1 book and 1 eraser, students may not say " 2 of something." Instead, they must say "1 book and 1 eraser."

5. Define and identify like and unlike terms in algebraic expressions.
   - Remind students that like means the same and unlike means different. Explain that in algebra, there are **like terms** and **unlike terms.**
   - Say, "Constants are always like terms." Ask students why they think constants are always like terms. (They are all numerals.)
   - Tell students that when terms include variables, the variable or variables must be identical to be considered like terms. Identical means that they must have the exact same letters and each must have the exact same exponent. For example, x and x are like terms; however x and x^2 are not like terms.
   - Tell students that the **coefficients** do not have to be the same to be like terms. Remind them that 3 erasers and 2 erasers would be 5 erasers. The erasers are the same, only the numbers are different. Therefore, 3x and 2x would be 5x. They are like terms because the variables are exactly the same. The coefficients don't matter.
   - Write the following list on the board. Discuss and then decide as a class whether each pair shown below includes like terms or unlike terms.

Like or Unlike Terms?

1) $12 + 20$	(Like, they are both constants.)
2) $9 + b$	(Unlike, there is a constant and a variable.)
3) $3x + x$	(Like, the variable is identical.)
4) $2p + 3r$	(Unlike, the variables are different.)
5) $n + n^2$	(Unlike, the variables have different exponents)
6) $6x^2 + 4x^2$	(Like, the variables are identical.)

   - Go back to the activity sheet Identifying Like and Unlike Terms and Simplifying.
   - Use the transparency copy of the activity sheet and colored overhead markers to model circling the like terms in a few problems. (Use the same color marker for all like terms.) (Note: The answer key has all like terms marked with the same shape. Where similar shapes are used, the thickness of the line has been changed - for example thin circles and thick circles.)
   - After completing several problems together, students should complete the remaining problems with markers or colored pencils. Allow them to discuss with a partner as they work.

6. Explore the concept of simplifying.
   - Write b + b on a blank transparency and hold up one book in each hand as you pose the following questions:
   - "If **b** stands for a book, then what does the expression b + b mean?" (1 book and 1 book) Remind students that when there is no coefficient in front of the variable, the coefficient is 1.
   - Ask, "How many **terms** are there in the expression b + b?" (2)
   - Ask, "Are they **like terms**?" (Yes)
   - Tell students that the expression b + b can be **simplified** because like terms can be **combined**. Say, "b + b means 1b plus 1b which equals 2b. The expression 2b is a simpler expression than b + b because it has fewer terms." Show that the expression b + b has 2 terms, but the expression 2b has only 1 term.)

7. Explore combining like terms.
   - Explain that **simplifying** an expression means to put all of the like terms together.
   - Write 2p + p on a clean transparency. Ask, "If **p** stands for pencil, what does the expression 2p + p mean?" (Model by holding up 2 pencils plus 1 pencil.)
   - Have students identify the variable and the operation. (The variable is p and the operation is addition.)
   - Ask, "How many terms are there?" (2)
   - Have students decide if they are like terms. (Yes)
   - Say, "Can the expression be simplified?" (Yes) "How can the expression be simplified?" (2p + p = 3p) Remind students that the expression 3p is simpler because it has fewer terms. (1)
   - Hold up1 book and 3 pencils. Ask students what expression may be written to represent the book plus the pencils. (b + 3p)
   - Have students identify the number of terms in the expression b + 3p. (2)
   - Have students decide if they are like terms? (No)
   - Ask, "Can the expression be simplified? Why?" (NO, because they are not **like terms**.)

8. Simplify expressions by combining like terms.
   - To reinforce the concept of simplifying like terms, write the expression a + 3 on the board and remind students that constants and variables are unlike terms so that a + 3 **cannot** be simplified. However, constants and other constants are like terms and can always be combined. For example, **a** + 2 + 1 can be simplified to a + 3.
   - Allow students to discuss how to simplify the following expressions. (It is not possible to simplify all of them.)

1) a + 2a	2) a + a + 3	3) a - b + c
3a	2a + 3	Not possible

4) $ab + 2ab + a + b$ $3ab + a + b$

5) $2b - 2b^2$ Not possible

- Using a clean transparency copy of the activity sheet Identifying Like and Unlike Terms and Simplifying, model how to combine like terms and simplify the first few expressions. Have students complete the worksheet and then compare answers with a partner.
- The activity sheets Working with Terms and Simplifying by Combining Terms are included for extra practice.

Language Development Activities

- Vocabulary Reinforcement

  The activity page Vocabulary Review will provide additional reinforcement of new terms introduced in this objective as well as review terms introduced earlier.

- Writing Prompt

  To reinforce the concepts in this lesson have students complete the writing activities in Parts III and IV of the Vocabulary Review activity page. Review the vocabulary words before students write sentences. Talk about real life meanings of the words also. For the writing prompt, encourage students to choose something they enjoyed doing in the lesson. Tell them to include the reasons why they liked that particular part of the lesson.

Additional Resources

The Algebra Lab, Middle School, Lesson 2 (Act. 3-4)

Name _____ Transparency / Student Copy

Identifying Like and Unlike Terms and Simplifying

1) $1 + 1 - 1 + 25$

2) $x + x + x$

3) $x + y + x - y$

4) $2x - x + y + 2y$

5) $x + 4 + y - 3$

6) $x^2 - y^2 + 2x + 2y$

7) $1 + x + 1 - 1 + xy$

8) $y^2 + 5x - 4 - y$

9) $6x + 8x$

10) $12w - w$

11) $4r + 4w - 3r$

12) $8y - 7y + 4$

13) $4a + 5b + 6a + 8c$

14) $4y + 3y + 12z + 5z$

15) $2xz + 7wz + 3xz$

16) $7n - 2n + 3n$

17) $6z + 1 + 2z$

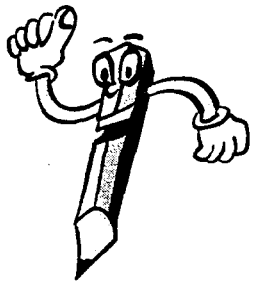
18) $a + 9m - n + 4x - 2$

19) $7y + 9y^2$

20) $19.3 + 17x - 2.3 + g$

Name: _____

Working with Terms



How many terms are in each expression?

1) $27 + 3 + 4b$ _____

2) $9y + 6y^2$ _____

3) $a + b + c + 3c$ _____

4) $5 + 5d + 2h + 1 + d^2$ _____

5) $100 + 100p$ _____

Do any of the terms have coefficients? Circle the coefficients.

1) $27 + 3 + 4b$

2) $9y + 6y^2$

3) $a + b + c + 3c$

4) $5 + 5d + 2h + 1 + d^2$

5) $100 + 100p$

There are <u>four</u> terms that have 1 as a coefficient.
Write the terms.

_____ _____ _____ _____

Put a (circle) around the like **constant** terms. Put a [box] around like **variable** terms.
If there are no like terms, write *unlike.*

1) $67 + y$

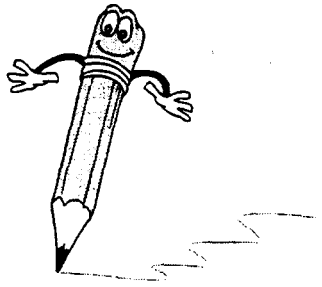
2) $a + 5a + 2a$

3) $4k + 27 + 4k^2$

4) $3 + 6b + 5 + b + 5b$

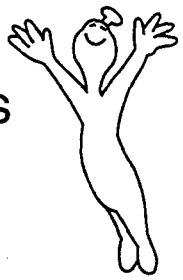
5) $h + h - 60 + h^2$

6) $99 - 9 + d + 9$



Name:_____

Simplify by Combining Terms



Simplify by combining like terms.

1) 3r - r _____

2) 10p - 9p _____

3) n + 4n _____

4) 8g + 20g _____

5) 6s - 5s + s _____

6) 8w + 9w - 7w _____

7) 6a - 7 - 5a + 7 _____

8) 10t - t + 11t _____

9) 3d + 4a + 5a _____

10) t + t + 3t _____

11) 7c - 4c + e + 5e _____

12) 3j + 2k +j + 4 + 3j + 10 _____

13) 5k + 5b - 4k _____

14) $4x^2 + 5x + 2 - 5x + 2$ _____

Simplify each expression. Circle the correct answer.

1) **4b + 2b + b** a) 8b b) 6b c) 7b d) $6b^3$

2) **5c - 2d - 3c** a) 0 b) 3c c) 2c + 2d d) 2c -2d

3) **8d + 4f - 8d** a) 4f b) 4d c) 16d +4f d) 4d + 4f

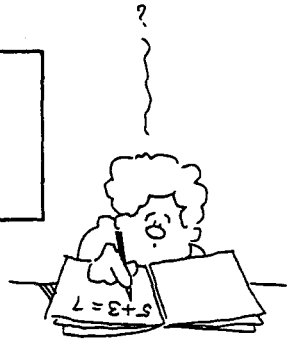
4) **5g + 3h -2h + 6g** a) 16gh b) 9g + 3h
c) 11g + h d) 11g + 5h

Name _____

Vocabulary Review

Part I. Sentence completions. Complete the sentences with a math term in the box. You will need to use one word twice.

coefficient	identical	operations
constant	like	simplify
expression	like term	unlike



1. An unlike term is the opposite of a(n)_____.
2. A variable is the opposite of a(n) _____.
3. The number in front of the variable is called the _____.
4. _____ terms have exactly the same variable.
5. _____ terms are different.
6. The number 7 in the expression $7b + 6$ is called the _____.
7. To _____ an expression, you can combine like terms.
8. 4x and 4x are _____ variables.
9. The four basic math _____ are addition, subtraction, multiplication, and division.
10. A(n) _____ includes variables, constants, and operations.

Part II. True or False. If the statement is **true**, write **T** on the line. If it is **false**, write **F**. If the statement is false, **correct** the statement to make it true.

_____ 1. The letter in front of the variable is called the coefficient.

_____ 2. Constants are like terms.

_____ 3. Variables are always unlike terms.

_____ 4. Variables must be identical to be considered unlike terms.

_____ 5. You can simplify an expression by combining unlike terms.

_____ 6. There are 2 terms in the expression $2 + 2 - 2$.

_____ 7. The number 2 in the expression $2x + 15$ is a constant.

_____ 8. Like terms must be identical.

_____ 9. An expression always includes numbers, operations, and variables.

_____ 10. You must substitute a number for a variable to evaluate an expression.

Part III. Writing Sentences. For each word listed, write **two** sentences. Write **one** sentence using the **math** meaning of the word. Write a **second** sentence using the **real life** meaning of the word.

1. **term**

_____

_____

2. **like**

_____

_____

3. **unlike**

_____

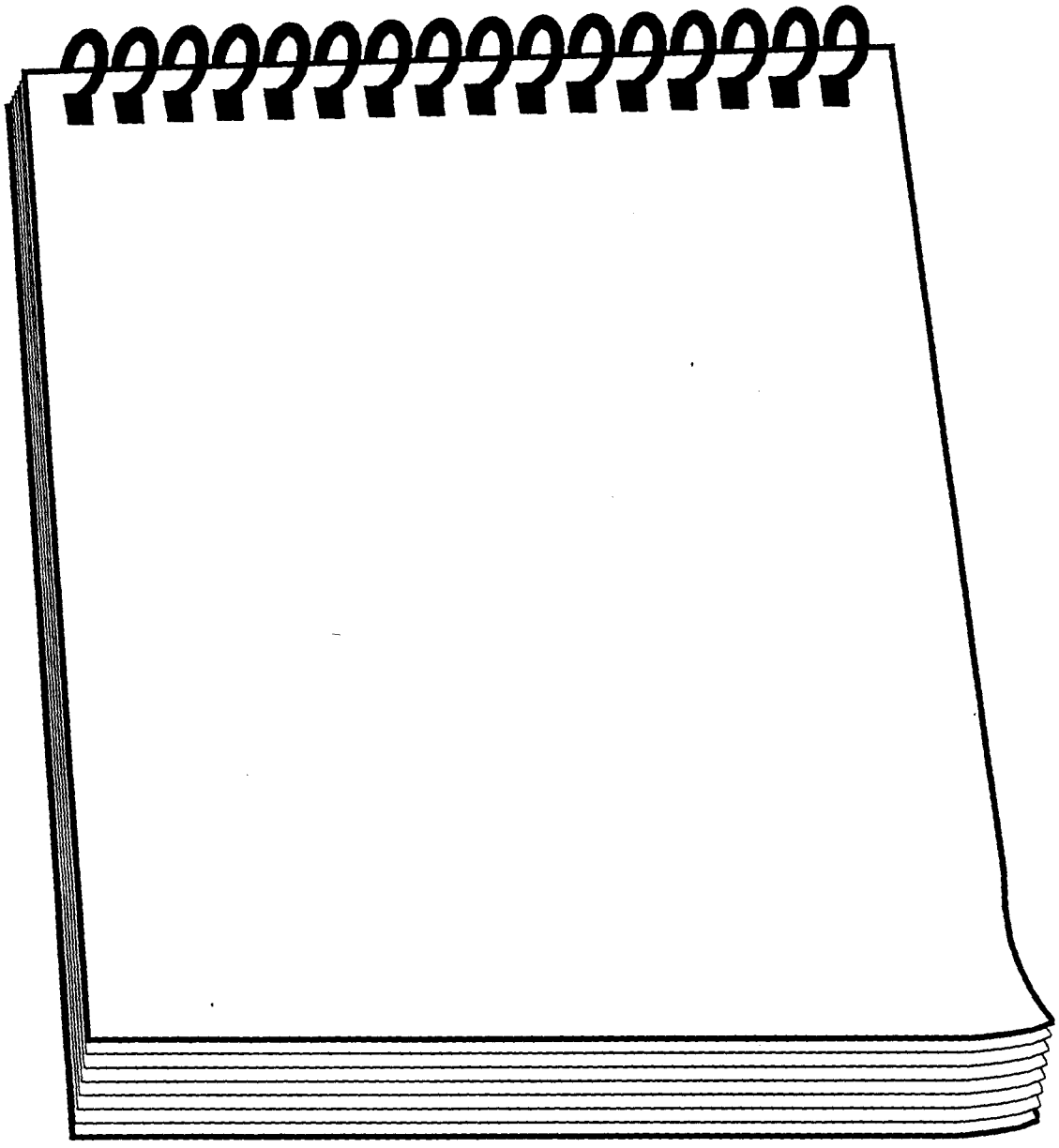
_____

4. **expression**

_____

_____

Part IV. Writing about Math. Write a letter to your teacher and tell him or her what you liked the most in this lesson. Tell why.



Answer Key
Obj. 6

Identifying Like and Unlike Terms and Simplifying

1) $1 + 1 - 1 + 25$

26

2) $x + x + x$

3x

3) $x + y + x + y$

2x

4) $2x - x + y + 2y$

x + 3y

5) $x + 4 + y - 3$

x + y +1

6) $x^2 - y^2 + 2x + 2y$

$x^2 - y^2 + 2x + 2y$

7) $1 + x + 1 - 1 + xy$

x + xy + 1

8) $y^2 + 5x - 4 - y$

$y^2 + 5x - y - 4$

9) $6x + 8x$

14x

10) $12w - w$

11w

11) $4r + 4w - 3r$

r + 4w

12) $8y - 7y + 4$

y + 4

13) $4a + 5b + 6a + bc$

10a + 5b + 8c

14) $4y + 3y + 12z + 5z$

7y + 17z

15) $2xz + 7wz + 3xz$

5xz + 7wz

16) $7n - 2n + 3n$

8n

17) $6z + 1 + 2z$

8z + 1

18) $a + 9m - n + 4x - 2$

9m + 4x + a - n - 2

19) $7y + 9y^2$

$9y^2 + 7y$

20) $19.3 + 17x - 2.3 + g$

17x + g + 17

Working with Terms

How many terms are in each expression?

1) 3
2) 2
3) 4
4) 5
5) 2

Circle the coefficients

1) **4**b
2) **9**y , **6**y^2
3) **3**c
4) **5**d , **2**h
5) **100**p

Terms with one as a coefficient

#3 - a b c ; #4 d^2

Like constaants and variables

1) unlike
2) a, 5a, 2a
3) unlike
4) 3, 5 ; 6b, b, 5b
5) h, h
6) 99, 9, 9

Simplify by Combining Terms

Simplify by combining like terms

1) 2r
2) p
3) 5n
4) 28g
5) 2s
6) 10w
7) a
8) 20t
9) 3d + 9a
10) 5t
11) 3c +6e
12) 7j + 2k + 14
13) k + 5b
14) $4x^2$ +4

Simplify Each Expression

1) c
2) d
3) a
4) c

Vocabulary Review

1) like term
2) constant
3) coefficient
4) like
5) unlike
6) coefficient
7) simplify
8) identical
9) operations
10) expression

True /False

1) T
2) T
3) F - sometimes
4) F - like
5) F - like
6) F - 3
7) F - 15 is a constant ; 2 is a coefficient
8) F (could have 2x + 5x)
9) F (usually)
10) T

Numerical Reasoning

Objective 7: Recognize and review estimation strategies. Choose the appropriate estimation strategy to fit a given situation.

Vocabulary

estimate
reasonable
front end
rounding

Materials

calculators

Transparencies

- Front End Estimation
- Estimating Using Front End
- Rules for Rounding
- Rounding

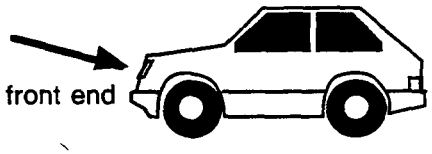
Student Copies

- Front End Estimation
- Practice with Front End Estimation
- Let's Round
- Rounding
- Practice with Rounding
- Estimation Review
- The Top 10
- Vocabulary Review

Language Foundation

1. Discuss the word **estimate** with students. Explain that the word can be a noun or a verb depending on how it is used. The pronunciation will change depending on whether it is a noun or a verb. As a ***noun*** estimate means a number close to an exact amount. An estimate tells about how much something is; it does not tell the exact amount. The ***verb*** estimate means to find a number close to the exact amount. Give students the common example of taking a car in to get it repaired. The mechanic estimates what he thinks it will cost to fix the car. The estimate is not exact, but close to the final cost.

2. Review the difference between front and back with students. Use a picture of a car and ask them to point out the **front end**. Write the number 527. Ask them to point out the "front end" of the number. (**5** 2 7)



3. Tell students that the number system in the U.S. uses ten **digits**, 0 - 9. The number 1,573 has four digits. Digits have place value. Give several example numbers and have students tell how many digits there are and give the place value of each.

4. Explain that the word **round** in English has several meanings. As an ***adjective***, round describes something that is shaped like a circle. As a ***verb*** round means to work with numbers using an estimation strategy called **rounding**.

Mathematics Component

1. Generate a discussion about estimating.
   - Ask the class how many students attend their school. Allow several students to make a "guess."
   - Ask each of the students to explain how they chose their answers. Some students will probably respond that they " just guessed." Others may have a method to share such as, "There are 6 teams of about 100 students per team so I guessed about 600 students."
   - Explain to the students that when they use reasoning or a specific method to make a guess this guess is called an **estimate.**
   - Explain that estimates can be used for a variety of reasons. Give students real life examples such as: estimate the total bill when shopping, estimate the amount of food needed for a party, and estimate the amount of time it will take to complete their homework. Explain that one of the best reasons to estimate is to check to see if answers are reasonable, do they make sense.

2. Investigate lead digit estimation.
   - Tell the students that they are going to review two methods of estimating, **front end** and **rounding**. Explain that each of these methods can be used with addition, subtraction, multiplication, or division.
   - Distribute student copies of the Front End Estimation activity sheet. Show the transparency Front End Estimation. Explain to students that when they use this method to estimate they will use only the front digit in each number. Explain that that is why this method of estimating is called **front end**.
   - Work through the examples together, discussing the points shown below. Have students copy the estimated numbers next to each problem on their papers as you write on the overhead. Note: The front digits must be in the same place value column. Having students draw a line through the front digits will help them remember this.

En. 1)
3,456 → 3,000 The front digit is a 3 in the thousand's column. Its value is 3,000.
+5,980 → +5,000 The front digit is a 5 in the thousand's column. Its value is 5,000.
8,000 The **front end estimate** when 3,000 and 5,000 are added is 8,000.

En. 2)
6,402 → 6,000 The front digit is a 6 in the thousand's column. Its value is 6,000.
- 751 → - 0,000 The front digit is a 0 in the thousand's column. Its value is 0,000.
6,000 The **front end estimate** when 0 is subtracted from 6,000 is 6,000.

En. 3)
342 → 300 The front digit is a 3 in the hundred's column. Its value is 300.
X270 → X200 The front digit is a 2 in the hundred's column. Its value is 200.
60,000 The **front end estimate** when 300 is multiplied by 200 is 60,000.

En. 4)
446 → 400 The front digit is a 4 in the hundred's column. Its value is 400.
269 → 200 The front digit is a 2 in the hundred's column. Its value is 200.
=2 The **front end estimate** when 400 is divided by 200 is 2.

- Exact answers for the examples above are as follows. 1) **9,436** 2) **5,651** 3) **92,340** 4) **1.658**

- Remind students that front end estimation is used to quickly tell about how much the answer to a problem will be. It will not be the exact answer. Front end estimation will always give a value less than the actual answer. Estimates can be used to check computation.
- A transparency/wall poster Estimating Using Front End is included for review.
- The activity sheet Practice With Front End Estimation is provided for further reinforcement.

3. Investigate rounding as a way to estimate.
   - Place the Rules for Rounding transparency/wall poster on the overhead. Review estimation rules as you discuss each step required to round 4,680 to the nearest hundred.
     - Locate and underline the digit you will round to. 4,6 8 0
     - Look at the digit to the right. 4,6 8 0
     - If the digit to the right is less than 5, round the hundreds digit down.
     - If the digit to the right is more than 5, round the hundreds digit up. 4,6 8 0 → 4,700
     - If the digit to the right is exactly 5, round the hundreds digit up.

     Note: Posting this set of rules in the classroom will give students a visual reference when rounding on their own.
   - Have students complete the activity sheet Let's Round!
   - Distribute student copies of the Rounding activity sheet. Place the Rounding transparency on the overhead. Work through the examples together, discussing the points shown below. Have students copy the rounding estimates on their papers as you write on the overhead.

Examples:

		Rounding Estimates	**Exact Answers**
	Nearest **ten thousand**:		
1)	49,632 →	50,000	49,632
	+ 9,897 →	+10,000	+ 9,897
		60,000	59,529
	Nearest **thousand**:		
2)	9,456 →	9,000	9,456
	− 6,980 →	− 7,000	− 6,980
		2,000	2,476
	Nearest **hundred**:		
3)	638 →	600	638
	x 84 →	x 100	x 84
		60,000	53,592

Nearest **ten**:

4) $801 \div 36 =$ ⟶ $800 \div 40 = 20$ $\qquad\qquad$ $801 \div 36 = 22.25$

- Have students complete Practice with Rounding for further reinforcement.

4. Explore whether front end or rounding is closer to an exact answer. Introduce the concept of a "reasonable" answer.
   - Give the students the following problem and ask them to estimate their answer using both methods of estimation.

368 + 497 = ?

Front End	**Rounding**
300	400
+ 400	+ 500
700	**900**

   - Have students find the sum of 368 and 497. (865)
   - Ask students which estimate is closer to the exact answer? (Rounding is closer to the exact answer.)
   - Say, "Why do you think estimation by rounding is closer to the exact answer?" (Answers will vary. Front end only uses the front digit to estimate, but rounding also uses information from other place value columns.)
   - Discuss the benefits of estimation with the students. Lead students to understand that estimation can help them check the reasonableness of their answers or can help them make decisions when an exact answer is not needed.
   - Explain to the students that estimation can even help when using a calculator. Write the problem 4,367 + 1,342 = 60,000 on the board. Tell the students to imagine that they are working this problem on a calculator and they get an answer of 60,000.
   - Ask if they think this is a **reasonable** answer. Say, "Reasonable means an answer makes sense. Does the answer 60,000 make sense for this problem?" (Students should answer no.)
   - Ask students why the answer does not make sense. (Front end and rounding give estimates of about 5,000. That estimate is very different from 60,000. 60,000 is far off from 5,000!)
   - Discuss what might cause students to get a wrong answer using a calculator. (For example, students may hit the wrong buttons or the calculator might not work right.) Lead students to see that estimation is a tool that can help them decide if answers are reasonable on paper or when using a calculator.
   - Give each student a copy of Estimation Review. Ask students to complete the activity page in class or as homework.

Language Development Activities

- Vocabulary Reinforcement

  The activity page Vocabulary Review will provide additional reinforcement of new terms introduced in this objective.

- Writing Prompt

  To reinforce the concept of estimation, have students complete "Writing about Math" in Part II of Vocabulary Review. Encourage students to include "real" examples with numbers in their response.

Front End Estimation

Examples:

		Lead Digit Estimates	Exact Answers
1)	$3,456 + 5,980$		$3,456 + 5,980$
2)	$6,402 - 751$		$6,402 - 751$
3)	342×270		342×270
4)	$\frac{446}{269} =$		$\frac{446}{269} =$

Estimating Using Front End

Remember: We use front end to help us estimate quickly.

To estimate a sum or difference using **front end**:

1) Find the **front** digits.
2) Add **zeros** to mark missing front digits and other place values.
3) **Add** or **subtrac**t the front digits.
4) Check to see if your answer is **reasonable**.

Estimate

$$\begin{array}{r} \boxed{2}\,4 \\ \boxed{5}\,1 \\ +\,\boxed{1}\,0 \\ \hline 8\,0 \end{array}$$

Front digit is 2.
Front digit is 5.
Front digit is 1.

Exact

$$\begin{array}{r} 2\,4 \\ 5\,1 \\ +\,1\,0 \\ \hline 8\,5 \end{array}$$

Front end estimate: 80

Estimate

$$\begin{array}{r} \boxed{3}\,4\,1 \\ 0\,2\,3 \\ +\,\boxed{0}\,0\,5 \\ \hline 3\,0\,0 \end{array}$$

Front digit is 3.
Fill in 0 as lead digit.
Fill in **two** us.

Exact

$$\begin{array}{r} 3\,4\,1 \\ 2\,3 \\ +\quad 5 \\ \hline 3\,6\,9 \end{array}$$

Front end estimate: 300

Estimate

$$\begin{array}{r} \boxed{7}\,2 \\ 0\,4 \\ +\,\boxed{2}\,3 \\ \hline 9\,0 \end{array}$$

Front digit is 7.
Fill in 0 as front digit.
Front digit is 2.

Exact

$$\begin{array}{r} 7\,2 \\ 4 \\ +\,2\,3 \\ \hline 9\,9 \end{array}$$

Front end estimate: 90

Estimate

$$\begin{array}{r} \boxed{8}\,7\,5 \\ -\,\boxed{4}\,0\,2 \\ \hline 4\,0\,0 \end{array}$$

Front digit is 8.
Front digit is 4.

Exact

$$\begin{array}{r} 8\,7\,5 \\ -\,4\,0\,2 \\ \hline 4\,7\,3 \end{array}$$

Front end estimate: 400

Name: _____

Practice with Front End Estimation

Remember: Add the front digits.
Write zeros for the other digits.
(For example, missing front digits and other place values)

Using front end estimation, get an estimated answer for each problem.
Then, use your calculator to find the exact answer.

Problem	Estimate	Exact Answer
1) 6,743 3,055 + 2,368		1) 6,743 3,055 + 2,368
2) 42 130 + 718		2) 42 130 + 718
3) 78,250 - 34,931		3) 78,250 - 34,931
4) 32 x87		4) 32 x87
5) 927 330		5) 927 330

Rules for Rounding

- Locate and underline the digit you will round to. 4,680
- Look at the digit to the right. 4,680
- If that digit is less than 5, round down. The number stays the same.
- If that digit is more than 5, round up.

  4,680 → 4,700

- If that digit is 5, round up.

Name:_____

Let's Round!

Round 1,572

to the nearest 10 } 1, 5 7 **2** (2<5 round down **1570**)

to the nearest 100 } 1, 5 **7** 2 (7>5 round up **1,600**)

to the nearest 1,000 } 1 , **5** 7 2 (5 = 5 round up **2,000)**

Round each number to the nearest **ten.**
(HINT: Underline the tens digit - draw an arrow to the digit on the right.)

1) 8 9 _____

2) 5 7 2 _____

3) 6,9 4 5 _____

4) 7 1 _____

5) 2 4 5 _____

6) 2 6 _____

Round each number to the nearest **hundred.**
(HINT: Underline the hundreds digit - draw an arrow to the digit on the right.)

1) 4 8 3 _____

2) 9 5 7 _____

3) 7, 6 7 1 _____

4) 1 3 2 _____

5) 3, 5 5 0 _____

6) 1 4, 3 2 1 _____

Round each number to the nearest **thousand**
(HINT: Underline the thousands digit - draw an arrow to the digit on the right.)

1) 1, 4 0 0 _____

2) 7 4 9 _____

3) 7, 5 0 0 _____

4) 5, 4 5 0 _____

5) 2 4, 7 9 9 _____

6) 6 2 4, 0 6 0 _____

Round **2 2, 6 8 6** to the nearest:

1) ten _____

2) hundred _____

3) thousand _____

4) ten thousand _____

Name: _____

Transparency/Student Copy

Rounding

Rounding Estimates	Exact Answers

Nearest ten thousand:

1) $\begin{array}{r} 49{,}632 \\ +\ 9{,}897 \\ \hline \end{array}$ $\begin{array}{r} 49{,}632 \\ +\ 9{,}897 \\ \hline \end{array}$

Nearest thousand:

2) $\begin{array}{r} 9{,}456 \\ -\ 6{,}980 \\ \hline \end{array}$ $\begin{array}{r} 9{,}456 \\ -\ 6{,}980 \\ \hline \end{array}$

Nearest hundred:

3) $\begin{array}{r} 658 \\ \times\ 84 \\ \hline \end{array}$ $\begin{array}{r} 658 \\ \times\ 84 \\ \hline \end{array}$

Nearest ten:

4) $801 \div 36 =$ $801 \div 36 =$

Name:_____

Practice with Rounding

Estimate using rounding. Then find an exact answer.

1) 581 x 8

2) 22 x 795

3) 709 + 450

4) 5, 206 + 2,715 + 643 + 185

5) 4) 156

6) 24,898 ÷ 5

7) 925 - 460

8) 14,045 - 3,750

9) The smallest school in Green County has 179 students.
The largest school in the county has 724 students.
About how many more students are in the largest school?

10) The stadium at Walker University has 3,479 seats.
The stadium at Jordan University has 9,870 seats.
About how many total seats are there at both universities?

11) A science kit costs $ 67.85 per student.
Mr. Push has $ 2,300 to spend for kits for his class of 37 students.
Does he have enough to buy the kits? Explain how you know.

Name: _____

Estimation Review

Estimate using **front-end** and **rounding**. Find the **actual answer**. Then, with your teacher, discuss the questions below.

Front- end		Rounding		Actual Answer
6,753 - 389	_____	6,753 - 389	_____	6,753 - 389
36 x 472	_____	36 x 472	_____	36 x 472

Which answer is closer to the actual answer? Why?

Is the exact answer greater or less than the front-end estimate? Why?

Is the exact answer greater or less than the estimate by rounding? Why?

Estimate by rounding. Circle the correct letter.

1) 5,324 + 942 + 3,890	a) 1,000	b) 5,000	c) 10,000	d) 15,000
2) 2,376 + 6,794 + 4,699	a) 14,000	b) 21,000	c) 28, 000	d) 35,000
3) 38 x 565	a) 24,000	b) 15,000	c) 20,000	d) 18,000
4) 2,812 x 11	a) 2,000	b) 11,000	c) 18,000	d) 28,000
5) 9,266 ÷ 33	a) 30	b) 200	c) 300	d) 3,000

Use front- end estimation. Circle the letter of the best estimate.

7,340
782
+ 6,029

a) less than 12,000
b) less than 13,000
c) greater than 14,000
d) greater than 15,000

$ 3.18
$ 2.32
+ $ 4.71

a) less than $7
b) less than $9
c) greater than $9
d) greater than $12

Use estimation to tell whether the statement is true or false.

1) 197 + 868 < 1,000 _____

2) 5,796 - 529 > 5,500 _____

3) 450 + 517 > 900 _____

4) 596 ÷ 3 < 200 _____

5) 2,883 ÷ 28 > 100 _____

6) 15, 073 ÷ 5 < 2,000 _____

Find the exact answer for only the problems with products less than 5,000.
(Hint: Estimate first, then do the computation.)

1) 4 x 858

2) 8 x 603

3) 894 x 6

4) 507 x 4

5) 9 x 1, 032

6) 450 x 5

The product will come between which two numbers? (Hint: Use estimation!)

Example: 3 x 4.8 Think: 4 x 3 = 12 Then, round 4.8 to 5. 3 x 5 = 15
So the product of 3 x 4.8 (14.4) will come between 12 and 15.)

1) 3 x 8.6 between _____ and _____

2) 8 x 4.59 between _____ and _____

4) 7 x 4.94 between _____ and _____

5) 5 x 5.72 between _____ and _____

Circle the letter of the best estimate:

421 + 2, 455 + 369

a) greater than 3,000
b) less than 3,000
c) greater than 2,000
d) less than 2,500

8,671 - 3,245

a) greater than 4,000
b) less than 4,000
c) greater than 5,000
d) less than 5,000

Name:_____ Problem Solving with Rounding

The Top 10

The table below shows the ten most populated states.
Use the table to answer the questions.

State	Population
California	33,145,000
Florida	15,111,000
Georgia	7,788,000
Illinois	12,128,000
Michigan	9,864,000
New Jersey	8,143,000
New York	18,197,000
Ohio	11,257,000
Pennsylvania	11,994,000
Texas	20,044,000

1) In what kind of order are the states listed?

_____

2) To what place value is each population rounded?

_____

3) Round each population to the nearest hundred thousand. Write your answers in the table. ⟶

State	Population
California	
Florida	
Georgia	
Illinois	
Michigan	
New Jersey	
New York	
Ohio	
Pennsylvania	
Texas	

RANK

1	6
2	7
3	8
4	9
5	10

4) Rank each state from greatest population (Rank 1) to least population (Rank 10)

⟵

Name _____

Vocabulary Review

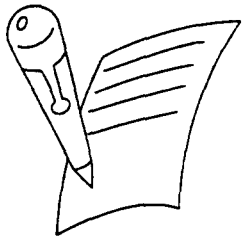
Part I. Sentence Completions. Use a word from the box to complete the sentences below.

digit	estimation	round
estimate	front end	rounding

1. When you _____ , you make a guess that is close to the exact answer.
2. _____ means you change a number to the nearest ten, hundred, thousand, ten thousand, etc.
3. A _____ is a number from 0 - 9.
4. One type of estimation is called _____ estimation.
5. You can _____ a number to the nearest ten, for example 26 → 30.
6. An _____ is a number close to the exact amount.

Part II. Writing about Math.

Write about an example in your everyday life when you would need to use **estimation** strategies. Describe the situation and explain the strategies you would use. Then, write about a time when you would need to have an **exact answer** instead of an estimate. Explain why an estimate would not be good enough.



Answer Key
Obj. 7

Practice with Front-End Estimation

Estimate	Exact Answer
1) 11,000	12,166
2) 800	890
3) 40,000	43,319
4) 2,400	2,784
5) 3	2.81

Let's Round

Nearest 10	Nearest 100	Nearest Thousand	Round 22,686
1) 90	1) 500	1) 1,000	1) 22,690
2) 570	2) 1,000	2) 1,000	2) 22,700
3) 6,950	3) 7,700	3) 8,000	3) 23,000
4) 70	4) 100	4) 5,000	4) 20,000
5) 250	5) 3,600	5) 25,000	
6) 30	6) 14,300	6) 624,000	

Practice with Rounding

Rounding - Exact Answer	
1) 4,800 (600 x 8) - 4,648	6) 5,000 (25,000 ÷ 5) - 4,979.6
2) 16,000 (20 x 800) - 17,490	7) 400 (900 - 500) - 465
3) 1,200 (700 + 500) - 1,159	8) 10,000 (14,000 - 4,000) - 10,295
4) 8,800 (5,000 + 3,000 + 600 + 200) - 8,749	9) 700 - 200 = 500
5) 40 (160 ÷ 4) - 39	10) 3,000 + 10,000 = 13,000
	11) $70 x 40 students = $2,800 No

Estimation Review

Front End	Round	Actual Answer
6,000 - 300 = 5,700	7,000 - 400 = 6,600	6,364

Rounding is closer because.place values other than just the first digit are taken into consideration. That will provide a more exact answer.

30 x 400 = 12,000	40 x 500 = 20,000	16,992

Rounding is closer.

The **front-end estimate** will be **less** than the exact answer because only the first digit of the largest place value is used. The exact answer, therefore, will be more than the estimate.

- With rounding, if you round all numbers **up**, the exact answer would be less than the estimate.
- If you round all numbers **down**, the exact answer would be more than the estimate.
- If you round some numbers up and some down, the exact answer could be more or less, depending upon the values rounded.

Estimate by Rounding

(Possible values for rounding are given. Values could vary depending on the place value students are rounding to.)

1) 5,000 + 900 + 4,000 = 9,900 (c)
2) 2,000 + 7,000 + 5,000 = 14,900 (a)
3) 40 x 600 = 24,000 (a)
4) 2,800 x 10 = 28,000 (d)
5) 9,000 ÷ 30 = 300 (c)

Use Front-End Estimation

7,000 + 6,000 (additional adjustment 700 + 300) = 14,000 (c)

3 + 2 + 4 = $9 (additional adjustment 70¢ + 30¢) = $10 (c)

True or False

1) 200 + 900 = 1,100 F
2) 5,800 - 500 = 5,300 F
3) 400 + 500 = 900 T
4) 600 ÷ 3 = 200 T
5) 2,800 ÷ 28 = 100 T
6) 15,000 ÷ 5 = 3,000 F

Problems with Products Less Than 5,000

1) 4 x 900 = 3,600 (3,432)
2) 8 x 600 = 4,800 (4,824)
3) 900 x 6 = 5,400 No
4) 500 x 4 = 2,000 (2,028)
5) 9 x 1,000 = 9,000 No
6) 500 x 5 = 2,500 (2,250)

Between Which Two Numbers

1) 3 x 8 = 24 3 x 9 = 27 (24 **25.8** 27)
2) 8 x 4 = 32 8 x 5 = 40 (32 **36.72** 40)
3) 7 x 4 = 28 7 x 5 = 35 (28 **34.58** 35)
4) 5 x 5 = 25 5 x 6 = 30 (25 **28.6** 30)

Circle the Letter of the Best Estimate

400 + 2,400 + 400 = 3,200 (a)

8,700 - 3,200 = 5,500 (c)

The Top 10

1) alphabetical order

2) thousands

3) See chart.

4) See chart.

State	Population
California	33,100,000
Florida	15,100,000
Georgia	7,800,000
Illinois	12,100,000
Michigan	9,900,000
New Jersey	8,100,000
New York	18,200,000
Ohio	11,300,000
Pennsylvania	12,000,000
Texas	20,000,000

RANK

1	California	6	Pennsylvania
2	Texas	7	Ohio
3	New York	8	Michigan
4	Florida	9	New Jersey
5	Illinois	10	Georgia

Vocabulary Review

1) estimate

2) rounding

3) digit

4) front-end

5) round

6) estimation

Possible answers to Writing about Math

Estimation could be used when speaking about population of states, towns, schools, number of people at a football game, time needed to complete a task or a trip, approximate cost of a list of groceries to see if you have enough money,etc.

An exact number would be needed for the number of players on a sports team (baseball must have 9 players), how much you actually have to pay for the groceries, how many fingers on two hands, the answer to a multiplication fact (3 x 10) , etc.

Objective 8: Find factors, multiples, primes, and composite numbers.

Vocabulary

multiples
infinity
least common multiple
factors
product
greatest common factor
prime numbers
composite numbers

Materials

calculators
markers

Transparencies

- Multiples
- Multiples and Factors Board
- Factors and Products
- The Factor Game
- Rules for Playing the Factor Game
- Finding Primes (Sieve of Eratosthenes)
- Factors, Multiples, Primes, and Composites

Student Copies

- All About Multiples
- Finding Factors
- The Factor Game
- Finding the Primes (Sieve of Eratosthenes)
- Review of Factors, Multiples,Primes, and Composites
- Vocabulary Review

Language Foundation

1. Ask students if they know the meaning of the word common. Explain that this word has more than one meaning in English. Tell students that they will be using the definition that means "shared by two or more things." Brainstorm examples of things that are common to students in the room. Have students make a list of supplies they bring to class. Ask students to share their lists and point out the items that are common to everyone. These would be items that appear on everyone's list such as a book bag, pencil, or book. Tell students that they will be looking at numbers in a similar way when they find the **least *common* multiples** and the **greatest *common* factors** in this lesson.

2. Tell students that the terms least and greatest are opposites. Explain that **least** means the "smallest number or amount" and **greatest** means the "largest number or amount."

3. To help students understand the concept of factors, discuss the definition of the word factor as something that can influence a situation. For example, if there is a big soccer game tomorrow and the forecast is for a bad rainstorm, this factor could cause the game to be canceled. Ask students to consider the factors they would use in purchasing a stereo such as price, size, quality of sound, and so on. Explain that in math, **factors** are numbers that multiply together to equal a **product**.

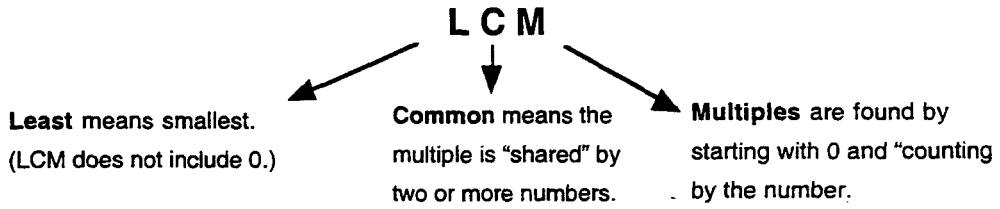
4. Point out that the verb compose means to combine together with other things to form something; for example, students put letters together to form words. Ask students to think of a composition. When they write a composition, they put together many parts. Introduce the word composite which means made up of different parts. Explain that in math a **composite** number has more than two factors.

Mathematics Component

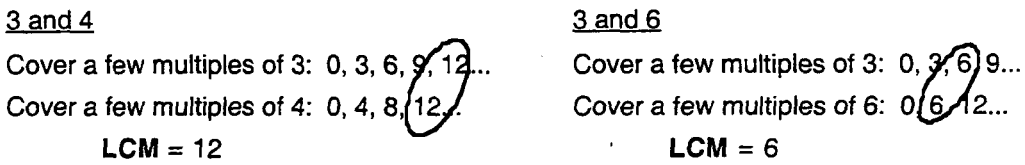
1. Introduce the concept of a multiple.
   - Write the following sequence on the board: 0, 2, 4, 6.... Ask a student to continue to count by twos as you add on to the list on the board. (Add six or seven more multiples.)
   - Point to the list and say, "These are **multiples** of 2." Write the word multiples. Explain that **multiples** of a number can be listed by starting with 0 and counting by the number.
   - Place the transparency Multiples on the overhead and review the definition of multiples given at the top. Explain that to find multiples of a number, students can multiply the number by 0, 1, 2, 3, 4... and so on. Go over the example and have students name other multiples of 3.
   - Also point out that students can "count by a number" to find multiples. (Students may be familiar with "skip counting" by twos, threes, fives, etc.)
   - Have students practice naming multiples of a few numbers, such as multiples of 4, 6, and 10.
   Note: The Multiples transparency may be enlarged to post for student reference.

2. Find the least common multiple (LCM) of two numbers. Introduce infinity.
   - Place the transparency Multiples and Factors Board on the overhead.
   - Have a student come up and cover several multiples of 2 with the same color transparent markers. (0, 2, 4, 6, 8, 10, 12...)
   - Ask if all of the multiples of 2 are covered with a marker? (No) Say, "How many multiples of 2 are there in all of the counting numbers?" Lead students to understand that since counting numbers go on forever, multiples of 2 go on forever, too. Explain that when something goes on forever we say that it goes on to **infinity**. Say, "**Infinity** means something goes on forever - it has no limits."
   - Have another student come up and cover several multiples of 3 with a second color of transparent markers. (0, 3, 6, 9 , 12, 15...)
   - Ask if all multiples of 3 are covered? (No) Say, "How many multiples of 3 are there in all of the counting numbers? (They go on forever to **infinity**.)
   - Write the word **common** on the board. Review the meaning of this word as described in the language foundation.
   - Have students look at the transparency with the colored markers. Ask, "Which multiples do 2 and 3 have in **common**? Name the common multiples of 2 and 3." (0,6, 12...) List the first few common multiples on the board. Be sure students understand that the multiples which are covered with two different colored markers are the ones which 2 and 3 have in common.
   - Remind students that the meaning of the word **least** is "the smallest." Point to the common multiples of 2 and 3 on the board and say, "What is the **least** common multiple of 2 and 3?" (0) Lead students to understand that 0 is a multiple of all numbers. Tell them that since 0 is a multiple of all numbers, we do not use it as a **least common multiple**. Have them look for the smallest common multiple which is not 0. (6) Say, "The least common multiple of 2 and 3 is 6."

- Write **LCM** of 2 and 3 is 6. Tell students that **LCM** stands for least common multiple. Review the meaning of each of the words in this phrase.

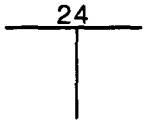


- Model using colored markers to find the least common multiple of other pairs of numbers. Begin by covering a few multiples of each number with different colored markers. If a common multiple other than 0 is not covered, add a few more to each set of multiples until a common multiple appears.



- Have students complete the activity sheet All About Multiples.

3. Introduce factors and products.
   - Place the transparency Factors and Products on the overhead.
   - Explain that **factors** of a number are numbers which multiply together to equal a product.
   - Point to the example 12 and explain that 1 and 12; 2 and 6; and 3 and 4 are numbers whose product is 12; therefore, they are **factors** of 12.
   - Also show students that factors of a number can be divided evenly into that number without a remainder. For example, 12 can be divided by 12, 6, 4, 3 , 2 and 1 with no remainder. Therefore, each of these is a factor of 12.
   - Write "5 is a factor of 45" on the board. Ask students how we know that 5 is a factor of 45. (45 can be divided by 5 with no remainder.) Say, "What are the other factors of 45? Hint: What numbers can 45 be divided by with no remainder?" (1, 3, 5, 9, 15, 45).
   - Ask, "What are the factors of 24?" Tell students that there are two ways to find the factors of 24. One way is to make a chart. Draw the following T-chart on the board and write 24 on the overhead as shown below.



   - Say, "Let's list all of the numbers which multiply together to equal the product 24." Tell students to start with 1 because 1 is always a factor. Ask students what number multiplied by 1

equals 24. (24) Model writing 1 and 24 on the chart as shown. Be sure to stress that 1 and 24 are both factors of 24 -- 1 is a factor and 24 is also a factor of 24.

24	
1	24

- Have students give other number pairs which multiply together to equal 24 and list them on the chart as shown. Then make a list to the side of all of the factors on the chart.

24	
1	24
2	12
3	8
4	6

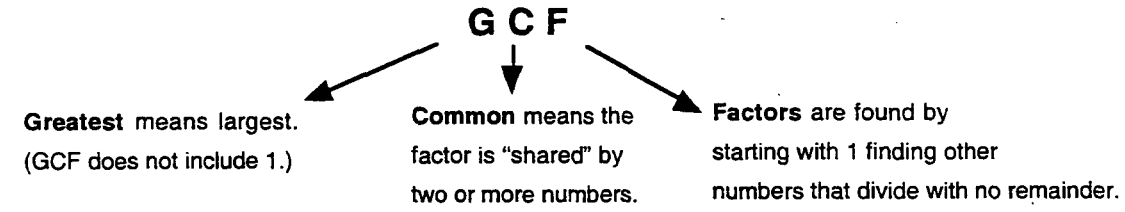
Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

- Tell students that another way to find the factors of 24 is to think about division. Ask students to think about what numbers divide into 24 without a remainder. Lead students to see that numbers should be tested in order (1, 2, 3 ,4, 5, 6...) to get a complete list of factors. Make a list of the factors on the board as each number is tested with division. When it is time to see if 24 can be divided by 6 without a remainder, show students that 6 is already on the list. **If students have checked** for factors in order, as soon as the factors begin to repeat, they have a complete list of factors.
- Say, "The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24."
- Distribute a Finding Factors activity sheet to each student. Do a couple of the problems with the class; then have students complete the first page of their activity sheet. This is a good activity for students to work on in pairs. Check answers together. Allow students to share their ideas on which method they find easiest for identifying factors and why.

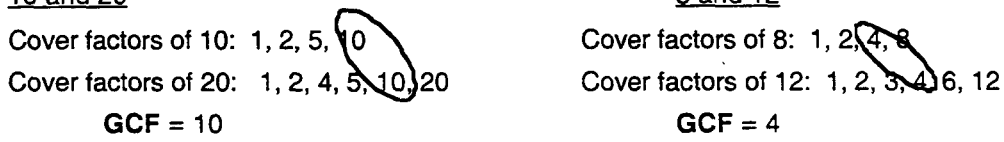
4. Introduce the concept of greatest common factor (GCF) of two numbers. This activity will be done in a similar way to the activity for finding the least common multiple of two numbers.
   - Place the transparency Multiples and Factors Board on the overhead.
   - Have a student come up and cover all of the factors (Not multiples!) of 6 with the same color transparent markers. (1, 2, 3, 6)
   - Have another student come up and cover all of the factors of 12 with a second color of transparent markers. (1, 2, 3, 4, 6, 12)
   - Review the meaning of the word **common**.
   - Have students look at the transparency with the colored markers. Ask, "Which factors do 6 and 12 have in **common**? Name the common factors of 6 and 12." (1, 2, 3, 6) List the common factors on the board. Be sure students understand that the factors which are covered with two different colored markers are the ones which 6 and 12 have in common.
   - Remind students of the meaning of the word **greatest** as "the largest." Point to the common

factors of 6 and 12 on the board and say, "What is the **greatest** common factor of 6 and 12?" (6) Lead students to understand that 1 is a factor of all numbers. Tell them that since 1 is a factor of all numbers, we do not use it as a **greatest common factor**. Have them look for the greatest common factor which is not 1. (6) Say, "The greatest common factor of 6 and 12 is 6."

- Write **GCF** of 6 and 12 is 6. Tell students that **GCF** stands for greatest common factor. Review the meaning of each of the words in this phrase.



- Model using colored markers to find the greatest common factor of other pairs of numbers. Begin by covering factors of each number with different colored markers. If a common factor other than 1 is not covered, there is no GCF.



- Have students complete the activity sheet Finding Factors.

5. Extend students' understanding of factors.
   - Tell students that they will play a game using factors.
   - Display the transparency The Factor Game.
   - Explain that you will demonstrate the rules of the game as you play one game against the class.
   - Point to the bottom of the page and write "teacher" above Player A and "class" above Player B.
   - Tell students that you will select a number from the game board at the top of the transparency.
   - Write the number under Player A (teacher) and cross the number off of the game board on the transparency with a colored marker. Tell students that this is your score at this time.
   - Tell students that now they have to work together to find as many factors of the number which you chose as possible, without using a calculator.
   - Ask the class to name all of the factors they have found. As they respond, record each of the factors under Player B (class) at the bottom of the transparency and cross them off of the game board. The sum of the factors is the score for the class at this time.
   - Explain that now the class may choose a number that has not been crossed off the game board. Cross the number off the game board with a different colored marker and add the number to the column labeled Player B (class). Explain that the sum of all of the numbers in that column is the score for the class at this time.

- Tell students that you will now find all of the possible factors that have **not already been crossed off** of the game board. Write them under Player A (teacher) and cross them off of the board. The sum of all of the numbers is now the teacher's new score.
- **NOTE:** If one team chooses a number which no longer has factors open on the board, say, "Oh no, this is an **illegal** move! This team loses a turn!" Go back to the other team and allow them to chose a number. **Remind students** that teams must always chose a number which leaves factors on the board for the other team - one team gets the number and the other team gets the factors.
- Play continues in this way until there are no moves left on the board. Find the sum for each of the teams.
- The team with the highest total score is the winner.
- Tell students that now they will play The Factor Game with a partner. Partners may want to use two different colored pencils to cross their numbers off the game board.
- Display the transparency Rules for Playing the Factor Game. Read the rules aloud.
- Distribute a clean copy of the activity sheet The Factor Game to each pair of students. Students may also want some extra paper to keep score on.
- Remind students that they will take turns making the first move and will play several games together, thinking about ways to get the best score.
- When all students have had a chance to play several games, allow them to discuss any winning strategies they have found with their partner. Then discuss ideas as a class.
- As students become better at playing this game, the game board can be expanded to 49 or students can be challenged to make their own game boards. Discussing winning strategies will promote higher level thinking and build oral language skills

6. Introduce prime and composite numbers.
   - Have students look back at their completed activity sheet Finding Factors.
   - Ask a student to name the factors of 13. (1 and 13).
   - Ask a student to name the factors of 29 (1 and 29).
   - Tell students that numbers that have exactly two factors are called **prime** numbers. Explain that 13 and 29 are **prime numbers.**
   - Ask, "Is 1 a prime number? Why?" (No. It has only one factor --1).
   - Say, "Is 12 a prime number? Why?" (No. It has more than two factors. 1, 2, 3, 4, 6, 12)
   - Ask students to talk with a partner to see if they can name other **prime numbers**? Have students share their ideas with the class. (2, 3, 5, 7,11...)
   - Ask students if they can name an even number that is prime. (2).
   - Say, "What are the **factors** of any **prime number**? (1 and the number itself).
   - Ask the class if 20 is a prime number. (No). Ask why. (It has factors of 2, 4, 5, 10).
   - Explain that numbers that have more than two factors are called **composite** numbers.

- Ask, "Is 35 prime or composite? (Composite). Why?" (It has factors of 1, 5 , 7, and 35).
- Have students give examples of some other composite numbers from their Finding Factors activity sheets. (6, 21, 35, 12, 9, 36, 40, 14, 72, and 100)

7. Identify prime numbers through 99.
   - Distribute individual copies of the activity sheet Finding Primes (Sieve of Eratosthenes). Students will also need markers or colored pencils. Place a transparency of the activity sheet on the overhead.
   - On the transparency, model drawing a box around the number 1 with a colored marker, because 1 is not a **prime** or a **composite** number. Have students do the same.
   - Have the students write a **P** on the number 2 with a different colored marker or pen. Explain that **P** means that it is a **prime** number. Remind students that it is a prime number because its only factors are 1 and 2.
   - Tell students to use the same marker or colored pencil to cross out all the **multiples** of 2 as you demonstrate on the overhead. Discuss with students different ways you could do this. (Count by two, cross out every other number, cross out any number that is even).
   - Ask if **P** should be written on the number 3. If they need a hint say, "Is 3 a prime number? (Yes). Have them write **P** on the number 3 using a different colored marker or colored pencil.
   - Then have them use the same colored pencil to cross out all of the **multiples** of 3 as you model on the overhead.
   - Ask if 4 is a prime number. (No because it has more than 2 factors - 1, 2, 4.)
   - Continue with this process until every number has been marked in some way. Some numbers will be crossed out more than once. This is okay. Tell students that all of the numbers marked with a **P** are **prime numbers**. Make a list of these prime numbers on the board and have students list them on paper.

     Prime Numbers Through 99

     (2, 3, 5, 7, 9, 11, 13, 17, 19, 23, 29,31,37, 41,43,47,53,59,61,67,71,73,77,79,83, 87, 89, 91, 97)

     Note: Have students keep this list for future reference.

   - A transparency Factors, Multiples, Primes, and Composites is provided for reinforcement and may be enlarged to use as a wall poster.
   - The activity sheets Review of Factors, Multiples,Primes, and Composites can be used to practice material presented in this lesson.

Language Development Activities

- Vocabulary Reinforcement

  The activity page Vocabulary Review will provide additional reinforcement of concepts and terms introduced in this lesson and review terms previously introduced.

Multiples

Multiples of a number are products of that number **and** another whole number.

Example: Multiples of 3

$3 \times 0 = 0$

$3 \times 1 = 3$

$3 \times 2 = 6$

$3 \times 3 = 9$

$3 \times 4 = ?$

(**HINT**: Count by a number to find multiples!)

Multiples of 5: 0, 5, 10, 15, 20 , 25 ...

Multiples and Factors Board

0	1	2	3	4	5	6	7	8	9	10
	11	12	13	14	15	16	17	18	19	20
	21	22	23	24	25	26	27	28	29	30
	31	32	33	34	35	36	37	38	39	40
	41	42	43	44	45	46	47	48	49	50
	51	52	53	54	55	56	57	58	59	60
	61	62	63	64	65	66	67	68	69	70
	71	72	73	74	75	76	77	78	79	80
	81	82	83	84	85	86	87	88	89	90
	91	92	93	94	95	96	97	98	99	100

Name_____

All About Multiples

Multiples of a number are products of that number **and** another whole number. List multiples of the following numbers.

Multiples of 6: ____ ____ ____ ____ ____ ____ ____.....

Multiples of 9: ____ ____ ____ ____ ____ ____ ____.....

Multiples of 10: ____ ____ ____ ____ ____ ____ ____.....

Find the least common multiple (LCM) of the following numbers.

L C M

Least *means smallest.* *(LCM does not include 0.)*

Common means the multiple is "shared" by two or more numbers.

Multiples are found by starting with 0 and "counting by the number.

EXAMPLE:

1) List the multiples. 2) Circle the common multiples. 3) Find the smallest or least common multiple.

Multiples of 2: 2, 4, 6, 8, (10), 12.....

Multiples of 5: 5, (10), 15, 20..... The **LCM** of 2 and 5 is **10**

8 : _ _ _ _ _ _ _ _

10: _ _ _ _ _ _ _ _

The LCM of 8 and 10 is _____

3 : _ _ _ _ _ _ _ _

5: _ _ _ _ _ _ _ _

The LCM of 3 and 5 is _____

8 : _ _ _ _ _ _ _ _

6: _ _ _ _ _ _ _ _

The LCM of 8 and 6 is _____

10 : _ _ _ _ _ _ _ _

25: _ _ _ _ _ _ _ _

The LCM of 10 and 25 is _____